Lecture 9

Database Normalization

COMP3278A

Introduction to Database Management Systems

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Outcome based learning

- Outcome 1. **Information Modeling**
  - Able to understand the modeling of real life information in a database system.
- Outcome 2. **Query Languages**
  - Able to understand and use the languages designed for data access.
- Outcome 3. **System Design**
  - Able to understand the design of an efficient and reliable database system.
- Outcome 4. **Application Development**
  - Able to implement a practical application on a real database.

Recap Armstrong's Axioms

- **We have 3 basic axioms.**
  - **1. Reflexivity - if $\beta \subseteq \alpha$, then $\alpha \rightarrow \beta$.**
  - **2. Transitivity - if $\alpha \rightarrow \beta$ and $\beta \rightarrow \gamma$, then $\alpha \rightarrow \gamma$.**
  - **3. Augmentation - if $\alpha \rightarrow \beta$, then $\gamma \alpha \rightarrow \gamma\beta$.**

Recap Armstrong's Axioms

- 3 more axioms to help easier prove!
  - **4. Union -** if $\alpha \rightarrow \beta$ and $\alpha \rightarrow \gamma$, then $\alpha \rightarrow \beta\gamma$.
  - **5. Decomposition -** if $\alpha \rightarrow \beta\gamma$, then $\alpha \rightarrow \beta$ and $\alpha \rightarrow \gamma$.
  - **6. Pseudo-transitivity -** if $\alpha \rightarrow \beta$ and $\gamma\beta \rightarrow \delta$, then $\alpha\gamma \rightarrow \delta$.
- 2 rules in tutorial!
  - **7. Extensivity -** if $\alpha \rightarrow \beta$, then $\alpha \rightarrow \alpha\beta$.
  - **8. Composition -** if $\alpha \rightarrow \beta$ and $\gamma \rightarrow \delta$, then $\alpha\gamma \rightarrow \beta\delta$.

Attribute set closure α^+

- Given a set F of FDs and a set of attributes α.
- The **closure of α** (denoted as α^+) is the set of attributes that can be **functionally determined by α**.

$$F = \{A \rightarrow B, B \rightarrow C\}$$

Attribute set closure of A. → $\{A\}^+ = \{ A, B, C \}$

1. A → A is always true (by **Reflexivity**).

2. A → B is given in F.

3. A → C is derived from F:
Given A → B and B → C, A → C is also true (by **Transitivity**).

Attribute set closure α^+

- Given a set F of FDs and a set of attributes α.
- The **closure of α** (denoted as **α^+**) is the set of attributes that can be **functionally determined by α**.

$$F = \{A \rightarrow B, B \rightarrow C\}$$

Attribute set closure of A.

$$\{A\}^+ = \{ A, B, C \}$$

$$\{B\}^+ = \{ B, C \}$$

$$\{C\}^+ = \{ C \}$$

$$\{A,B\}^+ = \{ A, B, C \}$$

Note that we only consider **single attribute**, not attribute sets (so we do not have AB, ABC, AC…etc in $\{A,B\}^+$).

FD closure F^+

- The set of **ALL functional dependencies** **that can be logically implied by F** is called the closure of F (or F^+)

- To compute $\mathbf{F^+}$ in a relation **R**:

This is the attribute set closure.

Step 1. Treat every subset of **R** as $\boldsymbol{\alpha}$.
Step 2. For every $\boldsymbol{\alpha}$, compute $\boldsymbol{\alpha}^+$.
Step 3. Use $\boldsymbol{\alpha}$ as LHS, and generate an FD for every subset of $\boldsymbol{\alpha}^+$ on RHS.

The **fd_closure()** algorithm

FD closure F^+

- Given a relation R(N, S, P) and the functional dependencies F = {**N→S**, **N→P**} find the FD closure F^+.

	N	S	P	NS	NP	SP	NSP

Step 1. Treat every subset of **R** as **α**.

FD closure F^+

- Given a relation R(N, S, P) and the functional dependencies F = {**N→S**, **N→P**} find the FD closure F^+.

	N	S	P	NS	NP	SP	NSP
Attribute set closure	{N,S,P}						

Step 2. For every **α**, compute **α**$^+$.

To find the attribute set closure {N}, use the attribute_closure() algorithm

1. ***result*** = {N}
2. Consider the FDs with **N →S**, **N →P**, add S and P into ***result.***
3. ***result*** = {N,S,P}

FD closure F^+

- Given a relation R(N, S, P) and the functional dependencies F = {**N→S**, **N→P**} find the FD closure F^+.

	N	S	P	NS	NP	SP	NSP
Attribute set closure	{N,S,P}	{S}	{P}	{N,S,P}	{N,S,P}	{S,P}	{N,S,P}

Step 2. For every **α**, compute $\boldsymbol{\alpha}^+$.

To find the attribute set closure {N}, use the attribute_closure() algorithm

1. ***result*** = {N}
2. Consider the FDs with **N →S**, **N →P**, add S and P into ***result.***
3. ***result*** = {N,S,P}

FD closure F^+

- Given a relation R(N, S, P) and the functional dependencies F = {N→S, N→P} find the FD closure F^+.

	N	S	P	NS	NP	SP	NSP
Attribute set closure	{N,S,P}	{S}	{P}	{N,S,P}	{N,S,P}	{S,P}	{N,S,P}
FD	N→N N→S N→P N→NS N→NP N→SP N→NSP						

Step 3. Use **α** as LHS, and generate an FD for every subset of α^+ on RHS.

FD closure F^+

- Given a relation R(N, S, P) and the functional dependencies F = {N→S, N→P} find the FD closure F^+.

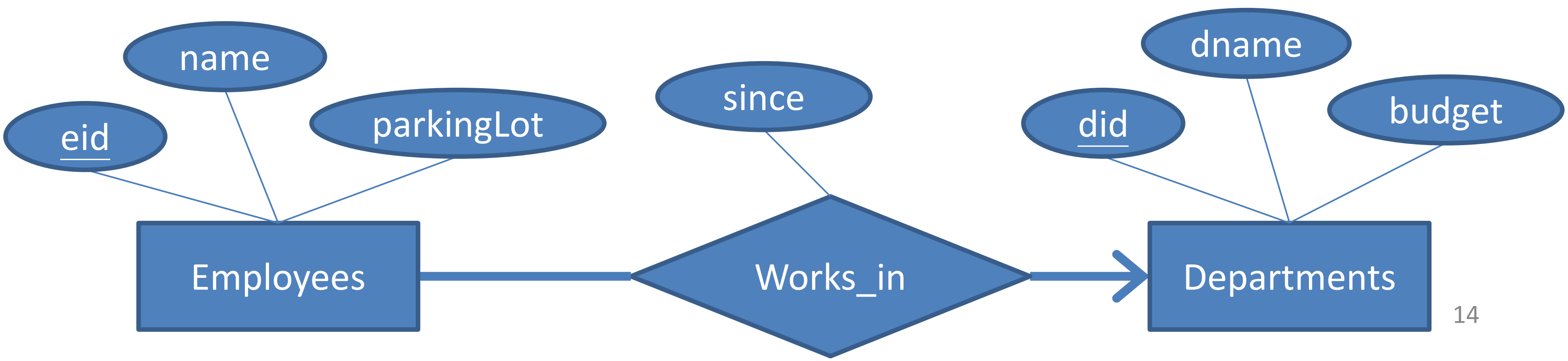
	N	S	P	NS	NP	SP	NSP
Attribute set closure	{N,S,P}	{S}	{P}	{N,S,P}	{N,S,P}	{S,P}	{N,S,P}
FD	N→N N→S N→P N→NS N→NP N→SP N→NSP	S→S	P→P	NS→N NS→S NS→P NS→NS NS→NP NS→SP NS→NSP	NP→N NP→S NP→P NP→NS NP→NP NP→SP NP→NSP	SP→S SP→P SP→SP	NSP→N NSP→S NSP→P NSP→NS NSP→NP NSP→SP NSP→NSP

Concept

- **Decomposition**
  - Lossless-join decomposition
  - Dependency preserving decomposition
- **Normal form**
  - Boyce-Codd Normal Form (BCNF)

Motivating example

- **Let's consider the following schema**
  - **Employees** have ***eid*** (key), ***name***, ***parkingLot***.
  - **Departments** have ***did*** (key), ***dname***, ***budget***.
  - An employee works in exactly one department, **since** some date.
  - Employees who work in the same department must park at the same ***parkingLot***.



Motivating example

- **Reduce to relational tables**
  - **Employees(** ***eid***, ***name***, ***parkingLot***, ***did***, ***since*** **)**

    Foreign key: ***did*** references Departments(did)
  - **Departments(** ***did***, ***dname***, ***budget*** **)**

Observation: In **Employees** table, whenever ***did*** is **1**, **parkingLot** must be "**A**"!
Implication: The constraint "*Employees who work in the same department must park at the same* ***parkingLot***" is **NOT** utilized in the design!!!
There are some **redundancy** in the Employees table.

eid	name	parkingLot	did	since
1	Kit	A	1	1/9/2014
2	Ben	B	2	2/4/2010
3	Ernest	B	2	30/5/2011
4	Betty	A	1	22/3/2013
5	David	A	1	4/11/2004
6	Joe	B	2	12/3/2008
7	Mary	B	2	14/7/2009
8	Wandy	A	1	9/8/2008

did	dname	budget
1	Human Resource	4M
2	Accounting	3.5M

Yes! As *parkingLot* is "**functionally depend**" on ***did***, we should not put ***parkingLot*** in the **Employee** table.

We are going to learn

- **Database normalization**
  - The process of organizing the columns and tables of a relational database to minimize redundancy and dependency.
- **To make sure that every relation $\boldsymbol{R}$ is in a “good” form.**
  - If $\boldsymbol{R}$ is not “good”, **decompose** it into a set of relations $\{\boldsymbol{R}_1, \boldsymbol{R}_2, \ldots, \boldsymbol{R}_n\}$.

Question: How can we do the decomposition?
Are there any guidelines / theories developed to decompose a relation?

Yes! The theories can be explained through **functional dependencies** ☺.

Normalization goal

- **We would like to meet the following goals when we decompose a relation schema R with a set of functional dependencies F into $R_1, R_2, \ldots, R_n$**
  - **1. Lossless-join** – Avoid the decomposition result in information loss.
  - **2. Reduce redundancy** – The decomposed relations R_i should be in **Boyce-Codd Normal Form** (**BCNF**).
  - **3. Dependency preserving** – Avoid the need to join the decomposed relations to check the functional dependencies when new tuples are inserted into the database.

Section 1

Lossless-join Decomposition

Example 1

R

A	B	C
1	1	3
1	2	2
2	1	3
3	2	2
3	1	3
4	2	2
4	1	3

Functional dependencies

F = {B →C}

The functional dependency **B→C** tells us that for all tuples with the same value in **B**, there should be **at most** one corresponding value in **C** (E.g., If B=1, C =3 ; if B=2, C=2)

Question: Will decomposing **R(A,B,C)** into **R_1(A,B)** and **R_2(A,C)** cause information lost?

Decompose

$R_1 = \pi_{A,\ B}(R)$

A	B
1	1
1	2
2	1
3	2
3	1
4	2
4	1

$R_2 = \pi_{A,\ C}(R)$

A	C
1	3
1	2
2	3
3	2
3	3
4	2
4	3

Think in this way:
Is this decomposition "**lossless join decomposition**"?
I.e., Is there any information lost if we decompose **R** in this way?

Example 1

R

A	B	C
1	1	3
1	2	2
2	1	3
3	2	2
3	1	3
4	2	2
4	1	3

Functional dependencies

F = {B → C}

$\neq$

$R_1 \bowtie R_2 = \pi_{A,\,B}(R) \bowtie \pi_{A,\,C}(R)$

A	B	C
1	1	3
1	1	2
1	2	3
1	2	2
2	1	3
3	2	2
3	2	3
3	1	2
3	1	3
4	2	2
4	2	3
4	1	2
4	1	3

Decompose

$R_1 = \pi_{A,\,B}(R)$

A	B
1	1
1	2
2	1
3	2
3	1
4	2
4	1

$R_2 = \pi_{A,\,C}(R)$

A	C
1	3
1	2
2	3
3	2
3	3
4	2
4	3

To check if the decomposition will cause information lost, let's try to join $\mathbf{R_1}$ and $\mathbf{R_2}$ and see if we can recover **R**.
As we see that $\mathbf{R_1} \bowtie \mathbf{R_2} \neq \mathbf{R}$, the decomposition has information lost.
This is <u>NOT a lossless-join decomposition</u>.

This is a bad decomposition

Example 2

R

A	B	C
1	1	3
1	2	2
2	1	3
3	2	2
3	1	3
4	2	2
4	1	3

Functional dependencies

$F = \{B \rightarrow C\}$

=

$R_1 \bowtie R_2 = \pi_{A,\,B}(R) \bowtie \pi_{B,\,C}(R)$

A	B	C
1	1	3
1	2	2
2	1	3
3	2	2
3	1	3
4	2	2
4	1	3

How about decomposing the relation **R(A,B,C)** into **R_1(A,B)** and **R_2(B,C)**?

Decompose

$R_1 = \pi_{A,\,B}(R)$

A	B
1	1
1	2
2	1
3	2
3	1
4	2
4	1

$R_2 = \pi_{B,\,C}(R)$

B	C
1	3
2	2

Well done! Since **$R_1 \bowtie R_2 = R$**, breaking down **R** to **R_1** and **R_2** in this way has no information lost. **This decomposition is a lossless-join decomposition.**

Lossless-join decomposition

R

A	B	C
1	1	3
1	2	2
2	1	3
3	2	2
3	1	3
4	2	2
4	1	3

Functional dependencies

$F = \{B \rightarrow C\}$

What is/are the condition(s) for a decomposition to be **lossless-join**?

Example I

NOT Lossless-join decomposition

$R_1 = \pi_{A,\,B}(R)$

A	B
1	1
1	2
2	1
3	2
3	1
4	2
4	1

$R_2 = \pi_{A,\,C}(R)$

A	C
1	3
1	2
2	3
3	2
3	3
4	2
4	3

Example II

Lossless-join decomposition

$R_1 = \pi_{A,\,B}(R)$

A	B
1	1
1	2
2	1
3	2
3	1
4	2
4	1

$R_2 = \pi_{B,\,C}(R)$

B	C
1	3
2	2

Lossless-join decomposition

R

A	B	C
1	1	3
1	2	2
2	1	3
3	2	2
3	1	3
4	2	2
4	1	3

Functional dependencies

F = {B → C}

1

A	B
1	1

Let's consider the first tuple (**1**,**1**,**3**) in **R**.

Note that there is only **ONE** tuple in $\mathbf{R_1}$ with **A=1, B=1**.

Example I

NOT Lossless-join decomposition

$R_1 = \pi_{A,\ B}(R)$

A	B
1	1
1	2
2	1
3	2
3	1
4	2
4	1

$R_2 = \pi_{A,\ C}(R)$

A	C
1	3
1	2
2	3
3	2
3	3
4	2
4	3

Lossless-join decomposition

R

A	B	C
1	1	3
1	2	2
2	1	3
3	2	2
3	1	3
4	2	2
4	1	3

Functional dependencies

$F = \{B \rightarrow C\}$

1

A	B
1	1

Let's consider the first tuple (**1**,**1**,**3**) in **R**.

Note that there is only **ONE** tuple in **R_1** with **A=1, B=1**.

2

A	C
1	3
1	2

Since **$A \rightarrow AC$** is **NOT** a functional dependency in F^+, there can be **more than one tuples** with **A=1** in **R_2** (e.g., (**1**,**3**), (**1**,**2**)) .

Example I

NOT Lossless-join decomposition

$R_1 = \pi_{A,\,B}(R)$

A	B
1	1
1	2
2	1
3	2
3	1
4	2
4	1

$R_2 = \pi_{A,\,C}(R)$

A	C
1	3
1	2
2	3
3	2
3	3
4	2
4	3

Lossless-join decomposition

R

A	B	C
1	1	3
1	2	2
2	1	3
3	2	2
3	1	3
4	2	2
4	1	3

Functional dependencies

F = {B → C}

Example I

NOT Lossless-join decomposition

$R_1 = \pi_{A, B}(R)$

A	B
1	1
1	2
2	1
3	2
3	1
4	2
4	1

$R_2 = \pi_{A, C}(R)$

A	C
1	3
1	2
2	3
3	2
3	3
4	2
4	3

1

A	B
1	1

Let's consider the first tuple (**1**,**1**,**3**) in **R**.

Note that there is only **ONE** tuple in $\mathbf{R_1}$ with **A=1, B=1**.

⋈

2

A	C
1	3
1	2

Since **A → AC** is **NOT** a functional dependency in F^+, there can be **more than one tuples** with **A=1** in $\mathbf{R_2}$ (e.g., (**1**,**3**), (**1**,**2**)) .

=

3

A	B	C
1	1	3
1	1	2

✔ (1, 1, 3)
✗ (1, 1, 2)

Therefore when we join $\mathbf{R_1}$ and $\mathbf{R_2}$, **more than one tuples will be generated** (i.e., (**1**,**1**) in $\mathbf{R_1}$ combine with (**1**,**3**) and (**1**,**2**) in $\mathbf{R_2}$)

Observation:

The decomposition of R(A,B,C) into R_1(**A**,B) and R_2(**A**,C) is NOT lossless-join because

- **A → AC**

is **NOT** in F^+ **, and … (to be explained in the next slide)**

Lossless-join decomposition

R

A	B	C
1	1	3
1	2	2
2	1	3
3	2	2
3	1	3
4	2	2
4	1	3

Functional dependencies

F = {B → C}

1

A	C
1	3

Let's consider the first tuple (**1,1,3**) in **R**.

Note that there is only **ONE** tuple in $\mathbf{R_2}$ with **A=1, C=3**.

Example I

NOT Lossless-join decomposition

$R_1 = \pi_{A,\ B}(R)$

A	B
1	1
1	2
2	1
3	2
3	1
4	2
4	1

$R_2 = \pi_{A,\ C}(R)$

A	C
1	3
1	2
2	3
3	2
3	3
4	2
4	3

Lossless-join decomposition

R

A	B	C
1	1	3
1	2	2
2	1	3
3	2	2
3	1	3
4	2	2
4	1	3

Functional dependencies

F = {B → C}

1

A	C
1	3

Let's consider the first tuple (**1,1,3**) in **R**.

Note that there is only **ONE** tuple in $\mathbf{R_2}$ with **A=1, C=3**.

2

A	B
1	1
1	2

Since **A → AB** is **NOT** a functional dependency in F^+, there can be **more than one tuples** with **A=1** in $\mathbf{R_1}$ (i.e., (**1,1**), (**1,2**)) .

Example I

NOT Lossless-join decomposition

$R_1 = \pi_{A,\ B}(R)$

A	B
1	1
1	2
2	1
3	2
3	1
4	2
4	1

$R_2 = \pi_{A,\ C}(R)$

A	C
1	3
1	2
2	3
3	2
3	3
4	2
4	3

Lossless-join decomposition

R

A	B	C
1	1	3
1	2	2
2	1	3
3	2	2
3	1	3
4	2	2
4	1	3

Functional dependencies

F = {B → C}

1

A	C
1	3

Let's consider the first tuple (**1,1,3**) in **R**.

Note that there is only **ONE** tuple in $\mathbf{R_2}$ with **A=1, C=3**.

⋈

2

A	B
1	1
1	2

Since **A → AB** is **NOT** a functional dependency in F^+, there can be **more than one tuples** with **A=1** in $\mathbf{R_1}$ (i.e., (**1,1**), (**1,2**)) .

=

3

A	B	C	
1	1	3	✔
1	2	3	✗

Therefore when we join $\mathbf{R_1}$ and $\mathbf{R_2}$, **more than one tuples will be generated** (i.e., (**1,3**) in $\mathbf{R_2}$ combine with (**1,1**) and (**1,2**) in $\mathbf{R_1}$)

Example I

NOT Lossless-join decomposition

$R_1 = \pi_{A,\,B}(R)$

A	B
1	1
1	2
2	1
3	2
3	1
4	2
4	1

$R_2 = \pi_{A,\,C}(R)$

A	C
1	3
1	2
2	3
3	2
3	3
4	2
4	3

Observation:

The decomposition of R(A,B,C) into R_1(**A**,B) and R_2(**A**,C) is NOT lossless-join because

- A→ AC (explained in previous slide), **and**
- **A→ AB**

are **NOT** in F^+ .

Lossless-join decomposition

R

A	B	C
1	1	3
1	2	2
2	1	3
3	2	2
3	1	3
4	2	2
4	1	3

Functional dependencies

F = {B → C}

1

A	B
1	1

Let's consider the first tuple (**1,1,3**) in **R**. Note that there is only **ONE** tuple in $\mathbf{R_1}$ with **A=1, B=1**.

Example II

Lossless-join decomposition

$R_1 = \pi_{A,\ B}(R)$ $R_2 = \pi_{B,\ C}(R)$

A	B
1	1
1	2
2	1
3	2
3	1
4	2
4	1

B	C
1	3
2	2

Lossless-join decomposition

R

A	B	C
1	1	3
1	2	2
2	1	3
3	2	2
3	1	3
4	2	2
4	1	3

Functional dependencies

F = {B →C}

1

A	B
1	1

Let's consider the first tuple (**1,1,3**) in **R**. Note that there is only **ONE** tuple in $\mathbf{R_1}$ with **A=1, B=1**.

2

B	C
1	3

Since **B →BC** is a functional dependency in F^+, there is only **one tuple** with **B=1** in $\mathbf{R_2}$.

Example II

Lossless-join decomposition

$R_1 = \pi_{A,\ B}(R)$ $R_2 = \pi_{B,\ C}(R)$

A	B
1	1
1	2
2	1
3	2
3	1
4	2
4	1

B	C
1	3
2	2

Lossless-join decomposition

R

A	B	C
1	1	3
1	2	2
2	1	3
3	2	2
3	1	3
4	2	2
4	1	3

Functional dependencies

F = {B → C}

1

A	B
1	1

Let's consider the first tuple (**1,1,3**) in **R**. Note that there is only **ONE** tuple in $\mathbf{R_1}$ with **A=1, B=1**.

⋈

2

B	C
1	3

Since **B → BC** is a functional dependency in F^+, there is only **one tuple** with **B=1** in $\mathbf{R_2}$.

=

3

A	B	C
1	1	3

✔

Therefore when we join $\mathbf{R_1}$ and $\mathbf{R_2}$, there will be **ONLY ONE tuple generated**, and that must be the corresponding tuple (**1,1,3**) in **R**.

Example II

Lossless-join decomposition

$R_1 = \pi_{A,\,B}(R)$ $R_2 = \pi_{B,\,C}(R)$

A	B
1	1
1	2
2	1
3	2
3	1
4	2
4	1

→

B	C
1	3
2	2

Observation:

The decomposition of R(A,B,C) into R_1(A,**B**) and R_2(**B**,C) is lossless-join because

- **B → BC**

is in F^+ .

Testing for lossless-join decomposition

- **Consider a decomposition of R into R_1 and R_2.**
  - Schema of R = schema of $R_1 \cup$ schema of R_2.
- **Let schema of $R_1 \cap$ schema of R_2 be R_1 and R_2's common attributes.**
  - A decomposition of R into R_1 and R_2 is lossless-join if and only if at least one of the following dependencies is in F^+ .

Schema of $R_1 \cap$ schema of $R_2 \rightarrow$ schema of R_1

OR

Schema of $R_1 \cap$ schema of $R_2 \rightarrow$ schema of R_2

Example

- **Question: Given R(A,B,C), F={B→C}, is the following a lossless join decomposition of R?**
  - R_1(A, B) , R_2(B, C)
- **Answer: To see if (R_1, R_2) is a lossless join decomposition of R, we do the following:**
  - Find common attributes of R_1 and R_2 : **B**
  - Verify if *any* of the FD below holds in F^+, if one of the FD holds, then the decomposition is lossless join.

    B → R_1 (i.e., B → AB?)

    B → R_2 (i.e., B → BC?) ✔
  - Since B → BC (by **Augmentation rule on** B→C), R_1 and R_2 are lossless join decomposition of R.

Section 2

Dependency preserving Decomposition

Dependency preserving

- **When decomposing a relation, we also want to keep the functional dependencies.**
  - A FD $X \rightarrow Y$ is preserved in a relation R if R contains all the attributes of X and Y.
- **If a dependency is lost when R is decomposed into R_1 and R_2:**
  - When we insert a new record in R_1 and R_2, we have to obtain $R_1 \bowtie R_2$ and check if the new record violates the lost dependency before insertion.
  - It could be very inefficient because joining is required in every insertion!

Dependency preserving

Note that A→CD is in F^+ because of the **Transitivity axiom.**

- **Consider R(A,B,C,D), F = {A → B, B →CD}**
  - F^+ = {A → B, B →CD, A →CD, trivial FDs}
- **If R is decomposed to R_1(A,B) , R_2(B,C,D):**
  - F_1 = {A → B, trivials}, the projection of F^+ on R_1
  - F_2 = {B → CD, trivials}, the projection of F^+ on R_2

R

A	B	C	D
1	1	3	4
2	1	3	4
3	2	2	3
4	1	3	4

Decompose

$R_1 = \pi_{A,\ B}(R)$

A	B
1	1
2	1
3	2
4	1

$R_2 = \pi_{B,\ C,\ D}(R)$

B	C	D
1	3	4
2	2	3

This is a **dependency preserving decomposition** as:

$$(F_1 \cup F_2)^+ = F^+$$

Let us illustrate the implication of dependency preserving in the next slide.

Dependency preserving

- **Consider R(A,B,C,D), F = {A → B, B → CD}**
  - F^+ = {A → B, B → CD, A → CD, trivial FDs}
- **Is this a lossless join decomposition?**
  - Yes! As B→R_2 (i.e., B→BCD) holds in F^+. That mean we can recover R by $R_1 \bowtie R_2$.
- **Why it is dependency preserving?**

Think about it…

If we insert a new record

A	B	C	D
5	1	4	4

into R_1 and R_2:

- R_1

A	B
5	1

- R_2

B	C	D
1	4	4

We need to check if the new record will make the database violate any FDs in F^+.

Is such decomposition allow us to do the validation on R_1 and R_2 **ONLY?** (But no need to join R_1 and R_2 to validate it?)

R

A	B	C	D
1	1	3	4
2	1	3	4
3	2	2	3
4	1	3	4

Decompose

$R_1 = \pi_{A,\,B}(R)$

A	B
1	1
2	1
3	2
4	1

$R_2 = \pi_{B,\,C,\,D}(R)$

B	C	D
1	3	4
2	2	3

Dependency preserving

- **F^+ = { A → B, B →CD, A →CD , trivials}**
  - Inserting tuple (5,1,4,4) violates B →CD.
- **The decomposition is dependency preserving as we only need to check:**
  - Inserting

A	B
5	1

    violate any F_1 in R_1?

    This involves checking F_1={A→B}.
  - Inserting

B	C	D
1	4	4

    violate any F_2 in R_2?

    This involves checking F_2={B →CD}.

We can check F_1 on R_1 and F_2 on R_2 only because $(F_1 \cup F_2)^+ = F^+$

R

A	B	C	D
1	1	3	4
2	1	3	4
3	2	2	3
4	1	3	4
5	1	4	4

Decompose

$R_1 = \pi_{A, B}(R)$

A	B
1	1
2	1
3	2
4	1
5	1

$R_2=\pi_{B, C, D}(R)$

B	C	D
1	3	4
2	2	3
1	4	4

Although among the two validations we haven't checked **A→CD**, but since A→B is checked in F_1, and B →CD is checked in F_2, if we pass both F_1 and F_2, it implies A →CD.

Dependency preserving

- **What about decompose R to R_1(A,B), R_2(A,C,D) ?**
- **R is decomposed to R_1(A,B) , R_2(A,C,D)**
  - $F^+ = \{A \rightarrow B,\ B \rightarrow CD,\ A \rightarrow CD$, trivial FDs$\}$
  - $F_1 = \{A \rightarrow B$, trivials$\}$, the projection of F^+ on R_1
  - $F_2 = \{A \rightarrow CD$, trivials$\}$, the projection of F^+ on R_2

This is **NOT a dependency preserving decomposition** as:

$$(F_1 \cup F_2)^+ \neq F^+$$

Let us illustrate the implication of NOT dependency preserving in the next slide.

R

A	B	C	D
1	1	3	4
2	1	3	4
3	2	2	3
4	1	3	4

Decompose

$R_1 = \pi_{A,\ B}(R)$

A	B
1	1
2	1
3	2
4	1

$R_2 = \pi_{A,\ C,\ D}(R)$

A	C	D
1	3	4
2	3	4
3	2	3
4	3	4

Dependency preserving

- **What about decompose R to $R_1(A,B)$, $R_2(A,C,D)$?**
- **Is this a lossless join decomposition?**
  - Yes! As $A \rightarrow R_1$ (i.e., $A \rightarrow AB$) holds in F^+. That mean we can recover R by $R_1 \bowtie R_2$.
- **Is it dependency preserving?**

R

A	B	C	D
1	1	3	4
2	1	3	4
3	2	2	3
4	1	3	4

Decompose

$R_1 = \pi_{A,\,B}(R)$

A	B
1	1
2	1
3	2
4	1

$R_2 = \pi_{A,\,C,\,D}(R)$

A	C	D
1	3	4
2	3	4
3	2	3
4	3	4

Think about it…

If we insert a new record

A	B	C	D
5	1	4	4

into R_1 and R_2:

- R_1

A	B
5	1

- R_2

A	C	D
5	4	4

We need to check if the new record will make the database violate any FDs in F^+. Is such decomposition allow us to do the validation on R_1 and R_2 **only** (but no need to join R_1 and R_2)?

Dependency preserving

- $F^+ = \{ A \rightarrow B, B \rightarrow CD, A \rightarrow CD \}$
  - Inserting tuple (5,1,4,4) **violates** $B \rightarrow CD$.
- **The decomposition is NOT dependency preserving as if we only check:**
  - Inserting

A	B
5	1

violate any F_1 in R_1?

This involves checking $F_1=\{A \rightarrow B\}$.

  - Inserting

A	C	D
5	4	4

violate any F_2 in R_2?

This involves checking $F_2=\{A \rightarrow CD\}$.

We CANNOT check F_1 on R_1 and F_2 on R_2 only because
$(F_1 \cup F_2)^+ \neq F^+$
Decomposition in this way requires joining tables to validate $B \rightarrow CD$ for **EVERY INSERTION**!

R

A	B	C	D
1	1	3	4
2	1	3	4
3	2	2	3
4	1	3	4
5	1	4	4

Decompose

$R_1 = \pi_{A, B}(R)$

A	B
1	1
2	1
3	2
4	1
5	1

$R_2=\pi_{A, C, D}(R)$

A	C	D
1	3	4
2	3	4
3	2	3
4	3	4
5	4	4

Although we passed F_1 and F_2, it doesn't mean that we passed all FDs in F!

It is because **we lost the FD $B \rightarrow CD$ in the decomposition**.

Dependency preserving

What is the condition(s) for a decomposition to be **dependency preserving**?

- **Let F be a set of functional dependencies on R.**
  - $R_1, R_2, \ldots, R_n$ be a decomposition of R.
  - F_i be the set of FDs in F^+ that include only attributes in R_i.
- **A decomposition is dependency preserving if and only if**

$$(F_1 \cup F_2 \cup \ldots \cup F_n)^+ = F^+$$

  - Where F_i is the set of FDs in F^+ that include only attributes in R_i.

Example 1

- **Given R(A, B, C) , F = {A → B , B → C}**
  - Is **R_1(A, B), R_2(B, C)** a dependency preserving decomposition?
- **First we need to find F^+ , F_1 and F_2.**
  - F^+ = {A→B , B→C, A→C, some trivial FDs}
  - F_1 = {A→B and trivial FDs }
  - F_2 = {B→C and trivial FDs }

Note that A→C is in F^+ because of the **Transitivity axiom.**

- **Then we check if $(F_1 \cup F_2)^+ = F^+$ is true.**
  - Since $F_1 \cup F_2$ = F ,this implies $(F_1 \cup F_2)^+ = F^+$.
- **This decomposition is dependency preserving.**

Example 2

- **Given R(A, B, C) , F = {A → B , B → C}**
  - Is **R_1(A, B), R_2(A, C)** a dependency preserving decomposition?
- **First we need to find F^+ , F_1 and F_2.**
  - F^+ = {A→B , B→C, A→C, some trivial FDs}
  - F_1 = {A→B and trivial FDs }
  - F_2 = {A→C and trivial FDs }

Note that A→C is in F^+ because of the **Transitivity axiom.**

- **Then we check if $(F_1 \cup F_2)^+ = F^+$ is true.**
  - Since B→C disappears in R_1 and R_2, $(F_1 \cup F_2)^+ \neq F^+$.
- **This decomposition is NOT dependency preserving.**

Section 3

Boyce-Codd Normal Form

FD and redundancy

- **Consider the following relation:**
  - **Customer(** ***id*, *name*, *dptID***)
  - F = { {***id***} → {***name***, ***dptID***} }

Customer

id	name	dptID
1	Kit	1
2	David	1
3	Betty	2
4	Helen	2

- **{*id*} is a key in Customer.**
  - Because the attribute closure of {***id***} (i.e., $\{id\}^+$ = {***id***, ***name***, ***dptID***}), which covers all attributes of **Customer**.

Observation: All non-trivial FDs in F form a key in the relation Customer.

- This implies that there are no other FD that is just involve a subset of columns in the relation.
- This implies that **Customer** has **no redundancy**.

FD and redundancy

- **As another example:**
  - **Customer(*id*, *name*, *dptID*, *building*)**
  - F = { {***id***} → {***name***, ***dptID*** , ***building***}
    {***dptID***} → {***building***} }

Customer

id	name	dptID	building
1	Kit	1	CYC
2	David	1	CYC
3	Betty	2	HW
4	Helen	2	HW

- **{*dptID*} → {*building*} brings redundancy. Why?**
  - Tuples have the same ***dptID*** must have the same ***building*** (e.g., ***dptID***=**1**, ***building***="**CYC**").
  - But those tuples can have different values in ***id*** and ***name***. For each different ***id*** values with the same ***dptID***, ***building*** will be repeated (**redundancy**).

For example, for tuples with (***id***=**1**, ***dptID***=**1**) and (***id***=**2**, ***dptID***=1) , ***building*** must equal "**CYC**" (redundancy).

FD and redundancy

- **As another example:**
  - **Customer(** ***id*, *name*, *dptID*, *building*** **)**
  - F = { {***id***} → {***name*, *dptID* , *building***}
    {***dptID***} → {***building***} }

Customer

id	name	dptID	building
1	Kit	1	CYC
2	David	1	CYC
3	Betty	2	HW
4	Helen	2	HW

- **How to check?**
  - Check if the attribute set closure of {***dptID***} covers all attributes in **Customer**. ({***dptID***}$^+$ = {***dptID, building***} ≠ **Customer**)

Redundancy is related to FDs. If there is an FD $\alpha \rightarrow \beta$, where $\{\alpha\}^+$ does not cover all attributes in R, then we will have redundancy in R!

Boyce-Codd Normal Form

- **Summarizing the observations, a relation R has no redundancy, or in Boyce-Codd Normal Form (BCNF), if the following is satisfied:**
  - For all FDs in $\mathbf{F^+}$ of the form $\alpha \rightarrow \beta$, where $\alpha \subseteq R$ and $\beta \subseteq R$, at least one of the following holds:

$\alpha \rightarrow \beta$ is trivial (i.e., $\beta \subseteq \alpha$)

We won't border with trivial FDs such as A→A, AB→A …etc

α is a key (superkey) for R

i.e., The attribute set closure of α, represented as $\{\alpha\}^+$, covers all attributes in **R**.

In other words, in BCNF, every non-trivial FD forms a key.

How to test for BCNF?

- **Formally, for verifying if R is in BCNF**
  - For each non-trivial dependency $\alpha \rightarrow \beta$ in $\mathbf{F^+}$ **(the functional dependency closure)**, check if α^+ covers the whole relation (i.e., whether α is a superkey).
  - If any α^+ does not cover the whole relation, **R** is not in BCNF.
- **Simplified test:**
  - It suffices to check only the dependencies in the given **F** for violation of BCNF, rather than check all dependencies in $\mathbf{F^+}$

For example, given R(A,B,C); F = {A→B, B→C},
we only need to check if both $\{A\}^+$ and $\{B\}^+$ cover {A,B,C}.
We do not need to derive F^+ = {A→B, B→C, A→C, …etc} and check each FD because A→C already considered when computing $\{A\}^+$.

How to test for BCNF?

- **However, if we decompose R into R_1 and R_2, we cannot use only F to check if the "decomposed" relations (i.e., R_1 and R_2) is BCNF, we have to use F^+ instead.**
- **Illustration**
  - R(A, B, C, D), F = {A → B, B → C}

To test if **R** is in BCNF, it suffices to check only the dependencies in **F** (but not **F^+**)

R

A	B	C	D
1	1	1	1
1	1	1	2
1	1	1	3
1	1	1	4
1	1	1	5

An example R that satisfies F

  - **$\{A\}^+$ covers all {A,B,C,D}?**
    Since $\{A\}^+$ = {A,B,C} ≠ {A,B,C,**D**}, **R** is not in BCNF.

As illustrated through this instance, since $\{A\}^+$ = {A,B,C} ≠ {A,B,C,**D**}, this implies that it will cause redundancy when we have tuples with the same value across {ABC} but different values in **D**.

How to test for BCNF?

To illustrate why we cannot use only **F** to test decomposed relations for BCNF, let's try to **decompose R into R_1(A, B) and R_2(A, C, D)**

- **Illustration**
  - R(A, B, C, D), F = {A → B, B → C}
- **Is R_2(A, C, D) in BCNF?**

R

A	B	C	D
1	1	1	1
1	1	1	2
1	1	1	3
1	1	1	4
1	1	1	5

When we check **R_2**, none of FDs in **F** is contained in **R_2**. Does this mean no non-trivial FDs are in **R_2**, and **R_2** is in BCNF?

R_1(A, B)

A	B
1	1

R_2(A, C, D)

A	C	D
1	1	1
1	1	2
1	1	3
1	1	4
1	1	5

No! We need to use **F^+** to verify if **R_2** is BCNF

How to test for BCNF?

- **In R_2(A, C, D), A→C is in F^+, because:**
  - **A→C** can be obtained by **transitivity rule on A→B and B→C**
  - There is a non trivial FD **A→C** in **R_2** that we have missed!
- **Therefore in R_2 we check $\{A\}^+ = \{A,C\} \neq \{A,C,D\}$**
  - Thus, **A** is not a key in R_2
  - **R_2** is NOT in BCNF.

R

A	B	C	D
1	1	1	1
1	1	1	2
1	1	1	3
1	1	1	4
1	1	1	5

Conclusion: *When we test whether a* ***decomposed relation*** *is in BCNF, we must project* ***F^+*** *onto the relation (e.g.,* ***R_2****), not* ***F****!*

R_1(A, B)

A	B
1	1

R_2(A, C, D)

A	C	D
1	1	1
1	1	2
1	1	3
1	1	4
1	1	5

Section 4

Normalization

Normalization goal

- **When we decompose a relation R with a set of functional dependencies F into R_1, R_2, …, R_n, we try to meet the following goals:**
  - **1. Lossless-join** – Avoid the decomposition result in information loss.
  - **2. No Redundancy** – The decomposed relations R_i should be in Boyce-Codd Normal Form (BCNF). (There are also other normal forms.)
  - **3. Dependency preserving** – Avoid the need to join the decomposed relations to check the functional dependencies.

Illustration

- **Consider R(A, B, C), F = {A→B , B→C}, is R in BCNF? If not, decompose R into relations that are in BCNF.**

R

A	B	C
1	1	2
2	1	2
3	1	2
4	1	2

- **Is R in BCNF?**
  - Because $\{B\}^+ = \{B,C\} \neq \{A,B,C\}$
  - Since $\{B\}^+$ does not cover all attributes in R, R is **NOT** in BCNF.

How should we decompose **R** such that the decomposed relations are always lossless join?
Note: A decomposition is lossless join if **at least one of the following dependencies is in F^+**

Schema of $R_1 \cap$ schema of $R_2 \rightarrow$ schema of R_1

OR

Schema of $R_1 \cap$ schema of $R_2 \rightarrow$ schema of R_2

Illustration

Idea: To make the decomposition always lossless join, we can pick the FD **A**$\rightarrow$**B** and make the decomposed relation as:

- R_1(**A**,**B**) – the attributes in the L.H.S. and R.H.S. of the FD.
- R_2(**A**,**C**) – the attribute(s) in the L.H.S. of the FD, and the remaining attributes that does not appear in R_1.

- If we decompose the relation R in this way the following must be true:

Schema of $R_1 \cap$ schema of $R_2 \rightarrow$ schema of R_1

- Schema of $R_1 \cap$ schema of R_2 is **A**.
- **A**$\rightarrow$**R**$_1$= **A**$\rightarrow$**AB must be true** because **R**$_1$ must consists of the L.H.S. and R.H.S. of the FD **A**$\rightarrow$**B** in **F**.

Illustration

	$R_1(A, B)$	$R_2(A, C)$
F_x	$A \rightarrow B$	$A \rightarrow C$

$F = \{A \rightarrow B, B \rightarrow C\}$ $F^+ = \{A \rightarrow B, B \rightarrow C, A \rightarrow C, \text{trivial FDs}\}$

- **Is $R_1(A, B)$ in BCNF?**
  - $F_1 = \{A \rightarrow B$, trivial FDs$\}$, it is a projection of F^+ on R_1.
  - Since $\{A\}^+ = \{A,B\} = R_1$, $\{A\}$ is a key in R_1.
  - Since all FDs in F_1 forms a key, R_1 is in BCNF.
- **Is $R_2(A, C)$ in BCNF?**
  - $F_2 = \{A \rightarrow C$, trivial FDs$\}$, it is a projection of F^+ on R_2.
  - Since $\{A\}^+ = \{A,C\} = R_2$, $\{A\}$ is a key in R_2.
  - Since all FDs in F_2 forms a key, R_2 is in BCNF.

R

A	B	C
1	1	2
2	1	2
3	1	2
4	1	2

R_1

A	B
1	1
2	1
3	1
4	1

R_2

A	C
1	2
2	2
3	2
4	2

Therefore, decomposing R(A, B, C) with $F = \{A \rightarrow B, B \rightarrow C\}$ to $R_1(A, B)$ and $R_2(A, C)$ result in a lossless join decomposition (**no information lost**), and BCNF relations (**no redundancy**)

Illustration

- **Is the decomposition dependency preserving ?**
  - $F = \{A \rightarrow B , B \rightarrow C\}$
  - $(F_1 \cup F_2) = (A \rightarrow B , A \rightarrow C)$
- Since $B \rightarrow C$ disappears in R_1 and R_2, $(F_1 \cup F_2)^+ \neq F^+$.
- The decomposition is **NOT dependency preserving**.

Note: Although the decomposition is not dependency preserving, but it is lossless join, so we can join $\mathbf{R_1}$ and $\mathbf{R_2}$ to test $B \rightarrow C$.

BCNF decomposition algorithm

result = {R};
done = **false**;
compute F^+;
while (**done** == **false**) {
    if (there is a schema R_i in result and R_i is not in BCNF)
        let $\alpha \rightarrow \beta$ be a non-trivial FD that holds on R_i s.t. $\{\alpha\}^+ \neq R_i$
        result = **(result** – R_i) $\cup$ (α β) $\cup$ (R_i – β)
    else
        done = **true**;
}

α is not a key;
$\alpha \rightarrow \beta$ causes R_i to violate BCNF

1. Delete R_i

2. Create a relation with only α and β

3. Create a relation containing R_i but with β removed.

Each R_i is in BCNF, and the decomposition must be lossless-join

Example 1

	R_1(B, C)	R_2(A, B)
F_x	B → C	A → B

- **Consider R(A, B, C), F = {A→B , B→C}, decompose R into relations that are in BCNF.**

Alternative decomposition: To make the decomposition always lossless join, we can pick the FD **B→C** and make the decomposed relation as:

- R_1(**B,C**) – the attributes in the L.H.S. and R.H.S. of the FD.
- R_2(**A,B**) – the attribute(s) in the L.H.S. of the FD, and the remaining attributes that does not appear in R_1.

R

A	B	C
1	1	2
2	1	2
3	1	2
4	1	2

R_1

B	C
1	2

R_2

A	B
1	1
2	1
3	1
4	1

Example 1

$F = \{A \rightarrow B, B \rightarrow C\}$ $F^+ = \{A \rightarrow B, B \rightarrow C, A \rightarrow C$, trivial FDs$\}$

	$R_1(B, C)$	$R_2(A, B)$
F_x	$B \rightarrow C$	$A \rightarrow B$

- **Decomposition: $R_1(B, C)$, $R_2(A, B)$**
- **Is $R_1(B, C)$ in BCNF?**
  - $F_1 = \{B \rightarrow C$, trivial FDs$\}$, it is a projection of F^+ on R_1.
  - Since $\{B\}^+ = \{B,C\} = R_1$, $\{B\}$ is a key in R_1.
  - Since all FDs in F_1 forms a key, R_1 is in BCNF.
- **Is $R_2(A, B)$ in BCNF?**
  - $F_2 = \{A \rightarrow B$, trivial FDs$\}$, it is a projection of F^+ on R_2.
  - Since $\{A\}^+ = \{A,B\} = R_2$, $\{A\}$ is a key in R_2.
  - Since all FDs in F_2 forms a key, R_2 is in BCNF.

R

A	B	C
1	1	2
2	1	2
3	1	2
4	1	2

R_1

B	C
1	2

R_2

A	B
1	1
2	1
3	1
4	1

Example 1

	$R_1(B, C)$	$R_2(A, B)$
F_x	$B \rightarrow C$	$A \rightarrow B$

- **Is the decomposition lossless join?**
  - From the illustration in example 1, the decomposition must be lossless join.
- **Is the decomposition dependency preserving ?**
  - $F = \{A \rightarrow B, B \rightarrow C\}$
  - $(F_1 \cup F_2) = (B \rightarrow C, A \rightarrow B)$
- Since $F = (F_1 \cup F_2)$, this implies $(F_1 \cup F_2)^+ = F^+$.
- **The decomposition is dependency preserving.**
  - That means if we insert a new tuple, if the new tuple does not violate F_1 in R_1, and F_2 in R_2, it won't violate F^+ in R.

R

A	B	C
1	1	2
2	1	2
3	1	2
4	1	2

R_1

B	C
1	2

R_2

A	B
1	1
2	1
3	1
4	1

Example 2

- **Consider a relation R in a bank:**
  R (*b_name*, *b_city*, *assets*, *c_name*, *l_num*, *amount*)
  - F = { {***b_name***} → {***assets***, ***b_city***},
    {***l_num***} → {***amount***, ***b_name***},
    {***l_num***, ***c_name***} → {all other attributes}}

Each specific value in ***bname*** is corresponds to at most one {***asset*** , ***b_city***} value

Each ***l_num*** corresponds to at most one {***amount***, ***b_name***} value.

Each { ***l_num***, ***c_name***} corresponds to at most one {***b_name***, ***b_city***, ***assets***, ***amount***} value.

- **Decomposition**
  - With {***b_name***} → {***assets***, ***b_city***}, $\{b\_name\}^+ \neq R$, R is not in BCNF.
  - Decompose R into **R_1**(***b_name***, ***assets***, ***b_city***) and **R_2**(***b_name***, ***c_name***, ***l_num***, ***amount***).

Example 2

- **Is R_1(b_name, assets, b_city) in BCNF?**
  - F_1 = { {***b_name***} → {***assets***, ***b_city***}, trivial FDs} ← Projection of F^+ on F_1.
  - {***b_name***}$^+$ = {***b_name***, ***assets***, ***b_city***} = R_1, so {***b_name***} is a key in R_1.
  - Since all FD in F_1 forms a key in R_1, R_1 is in BCNF.
- **Is R_2(b_name, c_name, l_num, amount) in BCNF?**
  - F_2 = { {***l_num***} → {***amount***, ***b_name***} ,
    {***l_num***, ***c_name***} → {all attributes} } ← Projection of F^+ on F_2.
  - {***l_num***}$^+$ = {***l_num***, ***amount***, ***b_name***} ≠ R_2, so {***l_num***} is NOT a key in R_2.
  - Since NOT all FD in F_2 forms a key in R_2, R_2 is NOT in BCNF.

Example 2

- **Picking {*l_num*} → {*amount, b_name*}, R_2 is further decomposed into:**
  - R_3(***l_num***, ***amount***, ***b_name***)
  - R_4(***c_name***, ***l_num***)
- **Is R_3(l_num, amount, b_name) in BCNF?**
  - F_3 = {{***l_num***} → {***amount***, ***b_name***}, trivial FDs}
  - {***l_num***}$^+$ = {***l_num***, ***amount***, ***b_name***} = R_3, so {***l_num***} is a key in R_3.
  - Since all FD in F_3 forms a key in R_3, R_3 is in BCNF.

Example 2

- **Is R_4(c_name, l_num) in BCNF?**
  - F_4 = {trivial FDs}
  - Since all FD in F_4 forms a key in R_4, R_4 is in BCNF.
- **Now, R_1, R_3 and R_4 are in BCNF;**
- **The decomposition is also lossless-join.**

Example 2

- **The decomposition is also dependency preserving.**
  - F_1 = { {***b_name***} → {***assets***, ***b_city***}, trivial FDs}
  - F_3 = {{***l_num***} → {***amount***, ***b_name***}, trivial FDs}

    {***l_num***} → {***b_name***} … (i)
    by **Decomposition of** {***l_num***} → {***amount***, ***b_name***}

    {***l_num***} → {***assets***, ***b_city***} … (ii)
    by **Transitivity of** (i) and {***b_name***} → {***assets***, ***b_city***}

    {***l_num***} → {***b_name*** ,***assets***, ***b_city, amount***} by **Union** of F_3 and (ii)

    {***l_num***, ***c_name***} → {***l_num*** ,***c_name***, ***b_name*** ,***assets***, ***b_city, amount***} by **Augmentation**

  - Therefore $F_1 \cup F_3 \cup F_4 = F$, which implies $(F_1 \cup F_3 \cup F_4)^+ = F^+$.
  - The decomposition is **dependency preserving.**

BCNF doesn't imply dependency preserving

- **It is not always possible to get a BCNF decomposition that is dependency preserving.**
- Consider R(A, B, C); F = { AB→C, C→B }
- There are two candidate keys: {AB}, and {AC}.
  - $\{AB\}^+ = \{A,B,C\} = R$
  - $\{AC\}^+ = \{A,B,C\} = R$
- R is not in BCNF, since **C** is not a key.
- Decomposition of **R** must fail to preserve AB→C.

R

A	B	C
1	1	2
2	1	2
1	2	3

R_1

A	B
1	1
2	1
1	2

R_2

B	C
1	2
2	3

Not lossless decomposition

R_1

A	B
1	1
2	1
1	1

R_2

A	C
1	2
2	2
1	3

Not lossless decomposition

R_1

A	C
1	2
2	2
1	3

R_2

B	C
1	2
2	3

lossless

$F_1 = \{\emptyset\}$

$F_2 = \{C \rightarrow B\}$

Not dependency preserving

Motivating example

- **Back to our motivating example, we have:**
  - **Employees(** ***eid***, ***name***, ***parkingLot***, ***did***, ***since***)
  - **Departments(** ***did***, ***dname***, ***budget***)
- "Employees who work in the same department must park at the same ***parkingLot***." implies the following FD:

FD: did → parkingLot

- **Is Employees in BCNF?**
  - {***did***}$^+$ = {***parkingLot***} ≠ {***eid***, ***name***, ***parkingLot***, ***did***, ***since***}
  - Since ***did*** is not a key, **Employees** is NOT in BCNF.

Normalization

- **Employees**(***eid***, ***name***, ***parkingLot***, ***did***, ***since***) is decomposed to
  - **Employees2**(***eid***, ***name***, ***did***, ***since***)
  - **Dept_Lots**(***did***, ***parkingLot***)
- With **Departments**(***did***, ***dname***, ***budget***), the above two decomposed relations are further refined to
  - **Employees2**(***eid***, ***name***, ***did***, ***since***)
  - **Departments**(***did***, ***dname***, ***parkingLot***, ***budget***)

Good design: parking lots for all employees can be updated by changing their department-specific ***parkingLot***.

Summary

- **Relational database design goals**
  - Lossless-join
  - No redundancy (BCNF)
  - Dependency preservation
- **It is not always possible to satisfy the three goals.**
  - A lossless join, dependency preserving decomposition into BCNF may not always be possible.
- **SQL does not provide a direct way of specifying FDs other than superkeys.**
  - Can use assertions to check FD, but it is quite expensive.

Question 1

Given a relation R(A,B,C) and a functional dependency {A→B} that holds on R, which of the following statements is/are correct?

a. {A,C} is a candidate key for R.
b. The decomposition of R into R_1(A,B) and R_2(A,C) is a lossless-join decomposition.
c. The decomposition of R into R_1(A,B) and R_2(B,C) is a lossless-join decomposition.

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Question: How to test if {A,C} is a **candidate key** of R or not?

Answer:
1) **[Superkey]** A candidate key is a superkey. If {A,C} is a superkey, its attribute closure $\{A,C\}^+$ must contains all attributes in R!
2) **[Minimal]** If {A,C} is minimal, it is a candidate key!

Is {A,C} a superkey?

1) Find attribute closure of A: $\{A\}^+$ = {A,B}. It is not a superkey.

2) Find attribute closure of C: $\{C\}^+$ = {C}. It is not a superkey.

3) Find attribute closure of {A,C}: $\{A,C\}^+$ = {A,B,C}. Since it contains all attributes of R, **it is a superkey.**

Is {A,C} minimal?

4) Since none of subset of {A,C} is a key, {A,C} is minimal, **it is a candidate key**.

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Question: What do we mean by lossless-join decomposition?

Answer: A decomposition is **lossless-join decomposition** if the original relation can be obtained after joining the decomposed relations.

Question: How to test if a decomposition is lossless-join decomposition?

Answer: A decomposition is lossless-join decomposition iff **at least one of the following dependencies is in F^+:**

1) common attribute of R_1 and R_2 → schema of R_1

2) common attribute of R_1 and R_2 → schema of R_2

1) Common attribute of R_1 and R_2:

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2) common attribute of R_1 and R_2 → schema of R_2

1) Common attribute of R_1 and R_2: **A**

2) **A →AB in F^+?** ✓

Since **A →B** , **A →AB** is true (Augmentation), **A →AB** in **F^+**!

3) Therefore, the decomposition is a lossless-join decomposition.

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1) Common attribute of R_1 and R_2:

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1) common attribute of R_1 and R_2 → schema of R_1

2) common attribute of R_1 and R_2 → schema of R_2

1) Common attribute of R_1 and R_2: **B**

2) **B →AB** in F^+? ✘

3) **B →BC** in F^+? ✘

4) Therefore, the decomposition is **NOT** a lossless-join decomposition.

Question 2 a

Consider the relation **R(A,B,C,D,E)** with the following functional dependencies **F = {A→E, BCE→D, CD→BE, C→AB, BD→AC}** which hold in it.

i) Which of the following attributes is NOT in the attribute closure of C (i.e., C^+)?

A. A.

B. B.

C. D.

D. E.

E. None of the above.

Question: What do we mean by the attribute closure of C?

Answer: The attribute closure of C is the set of attributes that can be functionally determined by C.

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Question: What do we mean by the attribute closure of C?

Answer: The attribute closure of C is the set of attributes that can be functionally determined by C.

Question: C→A holds?

1. Since C→AB,
2. **C→A** and C→B (decomposition)

Question 2 a

Consider the relation **R(A,B,C,D,E)** with the following functional dependencies **F = {A→E, BCE→D, CD→BE, C→AB, BD→AC}** which hold in it.

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A. A.
B. B.
C. D.
D. E.
E. None of the above.

Question: What do we mean by the attribute closure of C?

Answer: The attribute closure of C is the set of attributes that can be functionally determined by C.

Question: C→B holds?

1. Since C→AB,
2. C→A and **C→B** (decomposition)

Question 2 a

Consider the relation **R(A,B,C,D,E)** with the following functional dependencies **F = {A→E, BCE→D, CD→BE, C→AB, BD→AC}** which hold in it.

i) Which of the following attributes is NOT in the attribute closure of C (i.e., C^+)?

- ✔ A. A.
- ✔ B. B.
- ✔ C. D.
- D. E.
- E. None of the above.

Question: What do we mean by the attribute closure of C?

Answer: The attribute closure of C is the set of attributes that can be functionally determined by C.

Question: **C→D** holds?

Think in this way :

1. Since we have BCE→D **,** can we have **C→ BCE**?
2. We have C→B, therefore C→BC (augmentation), can we show that **C→E**?
3. Since C→A and A→E, **C→E** (transitivity)
4. Since C→BC and C→E, **C→BCE** (union)
5. Since C→BCE and BCE→D **,** C →D (transitivity)

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Consider the relation **R(A,B,C,D,E)** with the following functional dependencies **F = {A→E, BCE→D, CD→BE, C→AB, BD→AC}** which hold in it.

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3. Since C→A and A→E, **C→E** (transitivity)
4. Since C→BC and C→E, **C→BCE** (union)
5. Since C→BCE and BCE→D **,** C →D (transitivity)

Question 2 a

- Since C→AB,
- **C→A** and **C→B** (decomposition)
- Since C→A and A→E,
- **C→E** (transitivity)
- Since C→B and C→E,
- C→BE (union)
- Since C→BE,
- C→BCE (augmentation)
- Since BCE→D and C→BCE,
- **C→D** (transitivity)
- Hence, $C^+ = \{A,B,C,D,E\}$
- Answer: **E**

Question 2 b

Consider the relation **R(A,B,C,D,E)** with the following functional dependencies **F = {A→E, BCE→D, CD→BE, C→AB, BD→AC}** which hold in it.

- Which of the following functional dependencies is NOT in the closure of F (i.e., F^+)?
  - i. BE→D
  - ii. BD→ABCDE
  - iii. AB→D
  - iv. CE→A

Question: What do we mean by the FD closure?

Answer: The set of **all** functional dependencies that can be logically implied by F is called the closure of F (or F^+).

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Question: What do we mean by the FD closure?

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Think in this way :

1. Start from BD→AC , can we make R.H.S. more close to ABCDE?
2. BD→ABCD (augmentation)**, can we show that ABCD →ABCDE**?
3. Since A→E, we have ABCD→ABCDE (augmentation)
4. Therefore, **we can show that** BD→ABCDE **is true**!

Question 2 b

Consider the relation **R(A,B,C,D,E)** with the following functional dependencies **F = {A→E, BCE→D, CD→BE, C→AB, BD→AC}** which hold in it.

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✓ ii. BD→ABCDE
✗ iii. AB→D
✓ iv. CE→A

Question: What do we mean by the FD closure?

Answer: The set of **all** functional dependencies that can be logically implied by F is called the closure of F (or F^+).

Think in this way :

1. We have C→AB, and therefore we have C→A (decomposition); **can we show that CE→C ?**
2. **CE→C is always true due to reflexivity!**
3. Therefore, we have CE→C and C→A, we can show that CE→A is true (transitivity)!

Question 2 c

Consider the relation **R(A,B,C,D,E)** with the following functional dependencies **F = {A→E, BCE→D, CD→BE, C→AB, BD→AC}** which hold in it.

- Which of the following is/are candidate key(s) of R?
  i. {B,E}
  ii. {B,D}
  iii. {C,E}

Question: How to show if a set of attribute is a **candidate key** or not?

Question 2 c

Consider the relation **R(A,B,C,D,E)** with the following functional dependencies **F = {A→E, BCE→D, CD→BE, C→AB, BD→AC}** which hold in it.

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ii. { D}

iii. {C,

Question: How to show if a set of attribute is a **candidate key** or not?

Answer: 1) It has to be a super key. 2) It has to be minimal.

Question: Does $\{B,E\}^+$ covers all attributes in R? i.e. BE→ABCDE**?**

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✓ ii. {B,D}

iii.

Question: How to show if a set of attribute is a **candidate key** or not?

Answer: 1) It has to be a super key. 2) It has to be minimal.

Question: Does $\{B,D\}^+$ covers all attributes in R? i.e. BD→ABCDE**?**

Think in this way :

1. BD →BD must be true, therefore, we have to show
   1) BD→A, 2) BD→C, and 3) BD→E.
2. Since BD→AC**, BD→A and BD→C are true (decomposition)**
3. How about **BD→E**? We have A→E, can we show that BD→A?
4. BD→A already shown to be true, so now we have BD→A and A→E, therefore **BD→E is true!**

Question 2 c

Consider the relation **R(A,B,C,D,E)** with the following functional dependencies **F = {A→E, BCE→D, CD→BE, C→AB, BD→AC}** which hold in it.

- Which of the following is/are candidate key(s) of R?

✗ i. {B,E}

✓ ii. {B,D}

✗ iii. {C,E}

Question: How to show if a set of attribute is a **candidate key** or not?

Answer: 1) It has to be a super key. 2) It has to be minimal.

1. **Since we have already shown in part a) that** $\{C\}^+$covers all attributes of R, therefore C is a candidate key of R.
2. Hence, {C,E} must **NOT** be a candidate key of R because {C,E} is **not minimal**.

Question 2 d

Consider the relation **R(A,B,C,D,E)** with the following functional dependencies **F = {A→E, BCE→D, CD→BE, C→AB, BD→AC}** which hold in it.

- Suppose that relation R is decomposed into R_1(A,B,C,D) and R_2(A,C,E). Which of the following statements is/are correct?
  i. This is a lossless-join decomposition.
  ii. The decomposition is dependency preserving.
  iii. R_1 and R_2 are in BCNF.

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Consider the relation **R(A,B,C,D,E)** with the following functional dependencies **F = {A→E, BCE→D, CD→BE, C→AB, BD→AC}** which hold in it.

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✔ i. This is a lossless-join decomposition.

ii. The decomposition is dependency preserving.

iii. R_1 and R_2 are in BCNF.

Question: How to test if a decomposition is lossless-join decomposition?

Answer: A decomposition is lossless-join decomposition iff **at least one of the following dependencies is in F^+:**
1) common attribute of R_1 and R_2 → schema of R_1
2) common attribute of R_1 and R_2 → schema of R_2

1. Common attribute of R_1 and R_2 is **{A,C}.**
2. Test if any of the following FD are in F^+ :
    1. AC →ABCD
    2. AC →ACE
3. Since A→E **, AC →ACE is true** (augmentation).
4. Therefore, this is a lossless-join decomposition.

Question 2 d

Consider the relation **R(A,B,C,D,E)** with the following functional dependencies **F = {A→E, BCE→D, CD→BE, C→AB, BD→AC}** which hold in it.

- Suppose that relation R is decomposed into R_1(A,B,C,D) and R_2(A,C,E). Which of the following statements is/are correct?
  - ✔ i. This is a lossless-join decomposition.
  - ✔ ii. The decomposition is dependency preserving.
  - iii. R_1 and R_2 are in BCNF.

Question: How to test if a decomposition is dependency preserving or not?

Answer: A decomposition is dependency preserving iff:

$$(F_1 \cup F_2 \cup .... \cup F_n)^+ = F^+$$

Where F_i is the set of FDs in F^+ that include only attributes in R_i.

1. Determine F_1 and F_2, the projection of F^+ on R_1 and R_2
   1. **F_1 = {CD→B , C→ABD , BD→AC }.**
   2. **F_2 = { A→E , C→AE }.**
2. Although {**BCE→D, CD→BE**} are missing, but we know that {**C**}$^+$= **R**, so we have **C→ABD** in **F_1** and **C→AE in F_2**
   - **BCE→D and CD→BE** both have **C** on L.H.S., so these two FDs are in **$(F_1 \cup F_2)^+$**
3. Therefore **$(F_1 \cup F_2)^+ = F^+$**.
4. Therefore, this decomposition is dependency preserving.

Question 2 d

Consider the relation **R(A,B,C,D,E)** with the following functional dependencies **F = {A→E, BCE→D, CD→BE, C→AB, BD→AC}** which hold in it.

- Suppose that relation R is decomposed into R_1(A,B,C,D) and R_2(A,C,E). Which of the following statements is/are correct?

✔ i. This is a lossless-join decomposition.

✔ ii. The decomposition is dependency preserving.

iii. R_1 and R_2 are in BCNF.

Question: How to test if the decomposed relations are in BCNF?

Answer: **In BCNF, every non-trivial FD forms a key!**

Test if R_1 is in BCNF:

1. Determine F_1 , the projection of F^+ on R_1.
   1. **F_1 = {CD→B , C→AB , BD→AC , C→D}.**
2. Determine if all FDs in F_1 forms a key of R_1.
   1. Is {CD} a key in R_1? **Yes! Because $\{CD\}^+$={ABCD} in R_1, contains all attributes in R_1.**
   2. Is {C} a key in R_1? **Yes! Because $\{C\}^+$={ABCD} in R_1, contains all attributes in R_1.**
   3. Is {BD} a key in R_1? **Yes! Because $\{BD\}^+$={ABCD}, contains all attributes in R_1.**
3. Since **all non-trivial** FDs in F_1 forms a key, R_1 is in BCNF.

Question 2 d

Consider the relation **R(A,B,C,D,E)** with the following functional dependencies **F = {A→E, BCE→D, CD→BE, C→AB, BD→AC}** which hold in it.

- Suppose that relation R is decomposed into R_1(A,B,C,D) and R_2(A,C,E). Which of the following statements is/are correct?

✓ i. This is a lossless-join decomposition.

✓ ii. The decomposition is dependency preserving.

✗ iii. R_1 and R_2 are in BCNF.

Question: How to test if the decomposed relations are in BCNF?

Answer: **In BCNF, every non-trivial FD forms a key!**

Test if R_2 is in BCNF:

1. Determine F_2 , the projection of F^+ on R_2.
   1. **F_2 = {A→E , C→A}.**
2. Determine if all FDs in F_2 forms a key of R_2.
   1. Is {A} a key in R_2? **No! Because $\{A\}^+$={AE}, not covering all attributes in R_2.**
3. Since **not all non-trivial** FDs in F_2 forms a key, R_2 is NOT in BCNF.

Question 3

Consider the schema R(A, B, C, D, E) and the set of functional dependencies F={ D→A, C→BDE } which holds in the schema.

a) Find all candidate keys of R. Show your steps.

b) Give a lossless join decomposition of R into relations in BCNF. Is the decomposition dependency preserving?

Question 3

Consider the schema R(A, B, C, D, E) and the set of functional dependencies F={ D→A, C→BDE } which holds in the schema.

a) Find all candidate keys of R. Show your steps.

- Is C a candidate key?
  - Since C^+ is {A, B, C, D, E}, C is a **superkey**, and since C is **minimal**, it is a candidate key.
  - Since C is a candidate key, any supersets of C are not candidate keys.
- How about the combinations of other attributes?
  - C does not appear on the RHS of any non-trivial FD, therefore the value of C cannot be functionally determined by other attribute(s).
  - All combinations of other attributes (i.e. A,B,D, and E) cannot form a key. (Their attribute closure must not contain C)
- Thus, R has only one candidate key, which is C.

Question 3

Consider the schema R(A, B, C, D, E) and the set of functional dependencies F={ D→A, C→BDE } which holds in the schema.

b) Give a lossless join decomposition of R into relations in BCNF. Is the decomposition dependency preserving?

- R_1 (A,D), R_2 (B,C,D,E)
- **R1 in BCNF ?**
  - F_1 is the projection of F^+ on R_1: F_1 = {D→A, and the trivial FDs}
  - Is D a key in R_1?
  - Since D is a key in R_1, **all non-trivial FDs in F_1 forms a key, thus R_1 is in BCNF**.
- **R2 in BCNF?**
  - F_2 is the projection of F^+ on R_2: F_2 = {C→BDE, and the trivial FDs}
  - Is C a key in R_2?
  - Since C is a key in R_2, **all non-trivial FDs in F_2 forms a key, thus R_2 is in BCNF.**

Step 1. Decompose R into two relations R_1 and R_2, and make R_1 a BCNF.
Look at D→A in F, make **R_1 as (A,D).**
Then **R_2** is **(B,C,D,E).**
Why include D in R_2?

Step 2. Check if R_1 and R_2 are in BCNF. If not, further decompose it using Step 1.
In BCNF, every non-trivial FD forms a key!

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- **R2 in BCNF?**
  - F_2 is the projection of F^+ on R_2: F_2 = {C→BDE, and the trivial FDs}
  - Is C a key in R_2?
  - Since C is a key in R_2, **all non-trivial FDs in F_2 forms a key, thus R_2 is in BCNF.**
- **Is it a lossless-join decomposition?**
  - Common attribute among R_1 and R_2 = D
  - D→R_1 (i.e. AD) holds in F^+
  - Therefore it is a lossless-join decomposition.

Answer: A decomposition is lossless-join decomposition iff **at least one of the following dependencies is in F^+:**
1) common attribute of R_1 and R_2 → schema of R_1
2) common attribute of R_1 and R_2 → schema of R_2

Question 3

Consider the schema R(A, B, C, D, E) and the set of functional dependencies F={ D→A, C→BDE } which holds in the schema.

b) Give a lossless join decomposition of R into relations in BCNF. Is the decomposition dependency preserving?

- R_1 (A,D), R_2 (B,C,D,E)
- **R1 in BCNF ?**
  - F_1 is the projection of F^+ on R_1: F_1 = {D→A, and the trivial FDs}
  - Is D a key in R_1?
  - Since D is a key in R_1, **all non-trivial FDs in F_1 forms a key, thus R_1 is in BCNF**.
- **R2 in BCNF?**
  - F_2 is the projection of F^+ on R_2: F_2 = {C→BDE, and the trivial FDs}
  - Is C a key in R_2?
  - Since C is a key in R_2, **all non-trivial FDs in F_2 forms a key, thus R_2 is in BCNF.**
- **Is it a lossless-join decomposition?**
  - Common attribute among R_1 and R_2 = D
  - D→R_1 (i.e. AD) holds in F^+
  - Therefore it is a lossless-join decomposition.
- **Is it a dependency preserving decomposition?**
  - Since (F_1 U F_2)$^+$ = F^+, it is a dependency preserving decomposition.

Answer: A decomposition is dependency preserving iff:

$$(F_1 \cup F_2 \cup .... \cup F_n)^+ = F^+$$

Where F_i is the set of FDs in F^+ that include only attributes in R_i.

Question 4

Given R=(A, B, C, D), F={A→B, BC→D, D→A}

a) Find A^+ and C^+.

b) Find a candidate key of R.

c) Is B→D in F^+ ?

d) Is D→BC in F^+ ?

e) Is AC→D in F^+ ?

f) R is decomposed into R_1(A, B, C) and R_2(B, C, D). Is the decomposition lossless? Is the dependency preserving?

g) Is R in BCNF? If not, decompose R into relations in BCNF.

Question 4

Given R=(A, B, C, D), F={A→B, BC→D, D→A}

a) Find A^+ and C^+.

- A^+ = {AB}
- C^+ = {C}

b) Find a candidate key of R.

- **AC** or BC or CD

Question 4

Given R=(A, B, C, D), F={A→B, BC→D, D→A}

c) Is B→D in F^+ ?

- No. Because B^+ = {B}

d) Is D→BC in F^+ ?

- No. Because D^+ ={ABD}

e) Is AC→D in F^+ ?

- Yes. Because A→B and BC→D, we know that AC→D (pseudo-transitivity)

Question 4

Given R=(A, B, C, D), F={A→B, BC→D, D→A}

f) R is decomposed into R_1(A, B, C) and R_2(B, C, D). Is the decomposition lossless? Is the dependency preserving?

- Lossless?
  - $R_1 \cap R_2$=(B,C)
  - (B,C)→R_2
  - Hence, it is a lossless decomposition.

Question: How to test if a decomposition is lossless-join decomposition?

Answer: A decomposition is lossless-join decomposition iff **at least one of the following dependencies is in F^+:**

1) common attribute of R_1 and R_2 → schema of R_1

2) common attribute of R_1 and R_2 → schema of R_2

Question 4

Given R=(A, B, C, D), F={A→B, BC→D, D→A}

f) R is decomposed into R_1(A, B, C) and R_2(B, C, D). Is the decomposition lossless? Is the dependency preserving?

Answer: A decomposition is dependency preserving iff:

$$(F_1 \cup F_2 \cup \ldots \cup F_n)^+ = F^+$$

Where F_i is the set of FDs in F^+ that include only attributes in R_i.

- Lossless?
  - $R_1 \cap R_2$=(B,C)
  - (B,C)→R_2
  - Hence, it is a lossless decomposition.
- Dependency preserving?
  - F_1 = {A→B} ; F_2={BC→D, D → B}
  - Since (F_1U F_2)$^+$ does not equal to F^+ (D→A is not preserved), the decomposition is not dependency preserving

Question: How to test if a decomposition is dependency preserving or not?

Question 4

Given R=(A, B, C, D), F={A→B, BC→D, D→A}

g) Is R in BCNF? If not, decompose R into relations in BCNF.

- Is R in BCNF?
  - No. A→B and D→A violate BCNF as neither A nor D is a super key in R.

Question: How to test if the decomposed relations are in BCNF?

Answer: **In BCNF, every non-trivial FD forms a key!**

Question 4

Given R=(A, B, C, D), F={A→B, BC→D, D→A}

g) Is R in BCNF? If not, decompose R into relations in BCNF.

- If not, decompose R into relations in BCNF.
  - Decompose R into R_1(A, B) and R_2(A, C, D)
  - R_1 is in BCNF?
    - F_1 = {A →B}, A is a key in R1, **R_1 is in BCNF!**
  - R_2 is in BCNF?
    - F_2 = {D→A}, D is not a key in R_2, **R_2 is not in BCNF!**

Step 1. Decompose R into two relations R_1 and R_2, and make R_1 a BCNF.
Look at A→B in F, make **R_1 as (A,B).**
Then **R_2** is **(A,C,D).**

Step 2. Check if R1 and R2 are in BCNF. If not, further decompose it using Step 1.
In BCNF, every non-trivial FD forms a key!

Question 4

Given R=(A, B, C, D), F={A$\rightarrow$B, BC$\rightarrow$D, D$\rightarrow$A}

g) Is R in BCNF? If not, decompose R into relations in BCNF.

- If not, decompose R into relations in BCNF.
  - Decompose R into R_1(A, B) and R_2(A, C, D)
  - R_1 is in BCNF?
    - F_1 = {A $\rightarrow$B}, A is a key in R1, **R_1 is in BCNF!**
  - R_2 is in BCNF?
    - F_2 = {D$\rightarrow$A}, D is not a key in R_2, **R_2 is not in BCNF!**
  - We further decompose R_2 into R_3(A, D) and R_4(C, D).
  - R_3 is in BCNF?
    - F_3 = {D$\rightarrow$A}, D is a key in R_3, **R_3 is in BCNF!**
  - R_4 is in BCNF?
    - F_4 = {empty}, **R_4 is in BCNF!**
  - Hence, we can decompose R into **R_1, R_3 and R_4** in BCNF.