Lecture 8

Database Design: Functional Dependency

COMP3278A

Introduction to Database Management Systems

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Outcome based Learning

- Outcome 1. **Information Modeling**
  - Able to understand the modeling of real life information in a database system.
- Outcome 2. **Query Languages**
  - Able to understand and use the languages designed for data access.
- Outcome 3. **System Design**
  - Able to understand the design of an efficient and reliable database system.
- Outcome 4. **Application Development**
  - Able to implement a practical application on a real database.

Content

- **Important tools in database design.**
  - **Functional dependency.**
  - **FD closure.**
  - **Attribute set closure.**

What is FD?

- **Functional dependency (FD)** is a constraint between two sets of attributes in a relation from a database .
- It requires that the values of a certain set of attributes **uniquely determine (imply) the values for** another set of attributes.

$$X \rightarrow Y$$

Determine attribute set

Dependent attribute set

X → **Y** means that, for two tuples t_1 and t_2, if their values in **X** are the same, then their values in **Y** are also the same.

$$t_1[X] = t_2[X] \Rightarrow t_1[Y] = t_2[Y]$$

What is FD?

- Given a relation *R*, a set of attributes *X* in *R* is said to **functionally determine** another attribute *Y*, also in *R*, (written *X* → *Y*) if, and only if, each *X* value is associated with precisely one *Y* value.

{employee_id} → {name, phone} ✔

Employees

employee_id	name	phone
1	Jones	62225214
2	Smith	64459574
3	Parker	35564872
4	Smith	28975152

Important concept:
Primary key is just one of the FDs, we can have other FD constraints in the design of a database.

What is FD?

- Given a relation *R*, a set of attributes *X* in *R* is said to **functionally determine** another attribute *Y*, also in *R*, (written $X \rightarrow Y$) if, and only if, each *X* value is associated with precisely one *Y* value.

{employee_id} → {name, phone} ✔

Employees

employee_id	name	phone
1	Jones	62225214
2	Smith	64459574
3	Parker	35564872
4	Smith	28975152

{phone} → {name} ✔

In the company, each employee has his/her own phone number.
Therefore, the name attribute is functionally determined by the phone attribute.
Each phone number is associated with precisely one name .

What is FD?

- Given a relation *R*, a set of attributes *X* in *R* is said to **functionally determine** another attribute *Y*, also in *R*, (written $X \rightarrow Y$) if, and only if, each *X* value is associated with precisely one *Y* value.

{employee_id} → {name, phone} ✔

Employees

employee_id	name	phone
1	Jones	62225214
2	Smith	64459574
3	Parker	35564872
4	Smith	28975152

{phone} → {name} ✔

{name} → {phone} ✘

Question
Can you understand why this FD is not true?

What is FD?

- **Functional dependency is useful in database design.**
  - We can use FD to test if a **database instance** is legal.
  - We can specify constraints on the legality of **relation**.
  - It can help us design a better database (less redundancy)

Toy Example

R

A	B	C	D	E
1	5	2	5	4
1	4	3	2	3
3	4	3	2	2
4	2	4	1	4
4	1	4	1	4

A → B ✗

A → B is NOT true.

Reason:
These two tuples have the same value in **A**, but their values in **B** are not the same.

A	B
1	5
1	4
3	4
4	2
4	1

To check if **A** → **B** is satisfied in **R**, we have to check if the following condition is satisfied…
For all tuples in the instance, if their values in A are the same, then their corresponding values in B have to be the same.

Toy Example

R

A	B	C	D	E
1	5	2	5	4
1	4	3	2	3
3	4	3	2	2
4	2	4	1	4
4	1	4	1	4

A → B ✗

B → C ✔

B → C is true!

Notice:
It is fine if two tuples have different **B** values and have the same **C** value.

B	C
5	2
4	3
4	3
2	4
1	4

To check if **B** → **C** is satisfied in **R**, we have to check if the following condition is satisfied…
For all tuples in the instance, if their values in B are the same, then their corresponding values in C have to be the same.

Toy Example

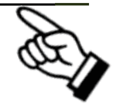
R

A	B	C	D	E
1	5	2	5	4
1	4	3	2	3
3	4	3	2	2
4	2	4	1	4
4	1	4	1	4

A → B ✗

B → C ✓

C → D ✓



C	D
2	5
3	2
3	2
4	1
4	1

To check if **C** → **D** is satisfied in **R**, we have to check if the following condition is satisfied…

For all tuples in the instance, if their values in C are the same, then their corresponding values in D have to be the same.

Armstrong's Axioms

R

A	B	C	D	E
1	5	2	5	4
1	4	3	2	3
3	4	3	2	2
4	2	4	1	4
4	1	4	1	4

A → B ✗

B → C ✓

C → D ✓

AB → A ✓

A	B
1	5
1	4
3	4
4	2
4	1

A
1
1
3
4
4

1. Reflexivity - if $\beta \subseteq \alpha$, then $\alpha \rightarrow \beta$.

Some FDs can be derived by rules. **Therefore we have the Armstrong's Axioms…**

1. Reflexivity: If RHS is a subset of LHS, then the FD must be true.

Obvious! This FD is ALWAYS TRUE!

Reason:

If two tuples have the same values on **AB**, then their **A** values must be the same!

Armstrong's Axioms

R

A	B	C	D	E
1	5	2	5	4
1	4	3	2	3
3	4	3	2	2
4	2	4	1	4
4	1	4	1	4

A → B ✗

B → C ✓

C → D ✓

AB → A ✓

B → D ✓

B	C	D
5	2	5
4	3	2
4	3	2
2	4	1
1	4	1

1. Reflexivity - if $\beta \subseteq \alpha$, then $\alpha \rightarrow \beta$.

2. Transitivity - if $\alpha \rightarrow \beta$ and $\beta \rightarrow \gamma$, then $\alpha \rightarrow \gamma$.

Again, this is obvious! Since B → C is true, and C → D is true.

This means that

1) if two tuples have the same B values, their C values must be the same.
2) if their C values are the same, their D values must be the same.

Therefore, B → D.

Armstrong's Axioms

R

A	B	C	D	E
1	5	2	5	4
1	4	3	2	3
3	4	3	2	2
4	2	4	1	4
4	1	4	1	4

A → B ✗

B → C ✓

C → D ✓

AB → A ✓

B → D ✓

AB → AD ✓

A	B
1	5
1	4
3	4
4	2
4	1

A	D
1	5
1	2
3	2
4	1
4	1

1. Reflexivity - if $\beta \subseteq \alpha$, then $\alpha \rightarrow \beta$.

2. Transitivity - if $\alpha \rightarrow \beta$ and $\beta \rightarrow \gamma$, then $\alpha \rightarrow \gamma$.

3. Augmentation - if $\alpha \rightarrow \beta$, then $\gamma\alpha \rightarrow \gamma\beta$.

Obvious!
Given B → D is true, we can derive that AB → AD is true!

Observation

Since **A** appears on both sides of the FD, whether the tuple values are the same will not be determined by **A.**

Armstrong's Axioms

R

A	B	C	D	E
1	5	2	5	4
1	4	3	2	3
3	4	3	2	2
4	2	4	1	4
4	1	4	1	4

A → B ✗

B → C ✓

C → D ✓

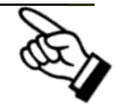
AB → A ✓

B → D ✓

AB → AD ✓

AC → CE ✓

A → E ✗



A	C
1	2
1	3
3	3
4	4
4	4

C	E
2	4
3	3
3	2
4	4
4	4

1. **Reflexivity - if $\beta \subseteq \alpha$, then $\alpha \rightarrow \beta$.**
2. **Transitivity - if $\alpha \rightarrow \beta$ and $\beta \rightarrow \gamma$, then $\alpha \rightarrow \gamma$.**
3. **Augmentation - if $\alpha \rightarrow \beta$, then $\gamma\alpha \rightarrow \gamma\beta$.**

IMPORTANT!!
Although **AC → CE** is true, can we derive **A → E** by augmentation? **NO!!**

Think in this way…

We cannot derived a tighter FD from a looser FD.

If we compare the tuples in **AC** and in **A**, there will be less tuples with the same values in **AC** than **A**. Therefore, **A → E** is a tighter FD than **AC → CE**.

Armstrong's Axioms

- We now have 3 basic axioms.
  - 1. Reflexivity - if $\beta \subseteq \alpha$, then $\alpha \rightarrow \beta$.
  - 2. Transitivity - if $\alpha \rightarrow \beta$ and $\beta \rightarrow \gamma$, then $\alpha \rightarrow \gamma$.
  - 3. Augmentation - if $\alpha \rightarrow \beta$, then $\gamma\alpha \rightarrow \gamma\beta$.
- New FDs can be derived (proved) using these axioms.

Question 1

Armstrong's axioms

1. **Reflexivity** - if $\beta \subseteq \alpha$, then $\alpha \rightarrow \beta$.
2. **Transitivity** - if $\alpha \rightarrow \beta$ and $\beta \rightarrow \gamma$, then $\alpha \rightarrow \gamma$.
3. **Augmentation** - if $\alpha \rightarrow \beta$, then $\gamma\alpha \rightarrow \gamma\beta$.

- Given a set of functional dependencies

$$F=\{ A \rightarrow B,\ B \rightarrow C,\ DE \rightarrow A \}.$$

- Prove, by **Armstrong's axioms** only, the following FDs are true.
  - **a)** $A \rightarrow C$.
  - **b)** $AD \rightarrow B$.
  - **c)** $DE \rightarrow ABC$.

Question 1a

Armstrong's axioms

1. **Reflexivity** - if $\beta \subseteq \alpha$, then $\alpha \rightarrow \beta$.
2. **Transitivity -** if $\alpha \rightarrow \beta$ and $\beta \rightarrow \gamma$, then $\alpha \rightarrow \gamma$.
3. **Augmentation -** if $\alpha \rightarrow \beta$, then $\gamma\alpha \rightarrow \gamma\beta$.

- Given a set of functional dependencies

$$F=\{ A \rightarrow B, B \rightarrow C, DE \rightarrow A \}.$$

- Prove, $A \rightarrow C$ is true.

Think in this way…
Since the target FD **starts from A** (i.e. $A \rightarrow C$), let see if we can find any existing FDs with LHS as A to start our prove.

Question 1a

Armstrong's axioms

1. **Reflexivity** - if $\beta \subseteq \alpha$, then $\alpha \rightarrow \beta$.
2. **Transitivity** - if $\alpha \rightarrow \beta$ and $\beta \rightarrow \gamma$, then $\alpha \rightarrow \gamma$.
3. **Augmentation** - if $\alpha \rightarrow \beta$, then $\gamma\alpha \rightarrow \gamma\beta$.

- Given a set of functional dependencies

$$F=\{ A \rightarrow B, B \rightarrow C, DE \rightarrow A \}.$$

- Prove, $A \rightarrow C$ is true.

$B \rightarrow C$ is true according to the problem definition!

Think in this way…

Now we choose $A \rightarrow B$ to start our prove.

Can we show that $B \rightarrow C$ is true such that we can prove $A \rightarrow C$ by **Transitivity**?

Question 1a

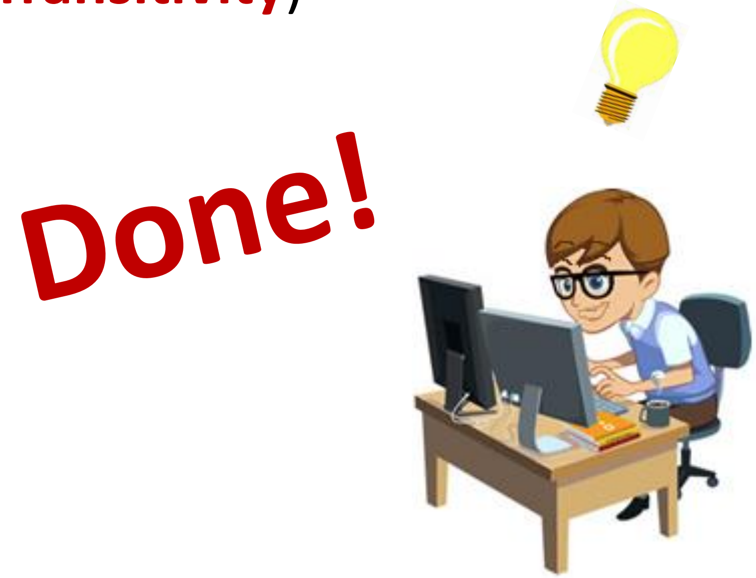
Armstrong's axioms

1. **Reflexivity** - if $\beta \subseteq \alpha$, then $\alpha \rightarrow \beta$.
2. **Transitivity** - if $\alpha \rightarrow \beta$ and $\beta \rightarrow \gamma$, then $\alpha \rightarrow \gamma$.
3. **Augmentation** - if $\alpha \rightarrow \beta$, then $\gamma\alpha \rightarrow \gamma\beta$.

- Given a set of functional dependencies

  F={ **A**→B, **B**→C, **DE**→A }.

- Prove, **A**→C is true.
  - Since **A**→B and **B**→C, A→C (by **Transitivity**)



Question 1

Armstrong's axioms

1. **Reflexivity** - if $\beta \subseteq \alpha$, then $\alpha \rightarrow \beta$.
2. **Transitivity** - if $\alpha \rightarrow \beta$ and $\beta \rightarrow \gamma$, then $\alpha \rightarrow \gamma$.
3. **Augmentation** - if $\alpha \rightarrow \beta$, then $\gamma\alpha \rightarrow \gamma\beta$.

- Given a set of functional dependencies

$$F=\{ A \rightarrow B,\ B \rightarrow C,\ DE \rightarrow A \}.$$

- Prove, by **Armstrong's axioms** only, the following FDs are true.
  - **a)** $A \rightarrow C$.
  - **b)** $AD \rightarrow B$.
  - **c)** $DE \rightarrow ABC$.

Question 1b

Armstrong's axioms

1. **Reflexivity** - if $\beta \subseteq \alpha$, then $\alpha \rightarrow \beta$.
2. **Transitivity** - if $\alpha \rightarrow \beta$ and $\beta \rightarrow \gamma$, then $\alpha \rightarrow \gamma$.
3. **Augmentation** - if $\alpha \rightarrow \beta$, then $\gamma\alpha \rightarrow \gamma\beta$.

- Given a set of functional dependencies

  $$F = \{ A \rightarrow B,\ B \rightarrow C,\ DE \rightarrow A \}.$$

- Prove, **AD** →**B** is true.
- **Think in this way…**
  - We have **A**→**B**, can we make it **AD**→**sth**?

    If **A**→**B**, then **AD**→**BD** (by **Augmentation**)
  - We have **AD**→**BD** , can we have **BD**→**B**?

    BD→**B** is always true because of **Reflexivity**!
  - So we now have **AD**→**BD** and **BD**→**B**, so **AD** →**B by transitivity** !!!

Question 1b

Armstrong's axioms

1. **Reflexivity** - if $\beta \subseteq \alpha$, then $\alpha \rightarrow \beta$.
2. **Transitivity** - if $\alpha \rightarrow \beta$ and $\beta \rightarrow \gamma$, then $\alpha \rightarrow \gamma$.
3. **Augmentation** - if $\alpha \rightarrow \beta$, then $\gamma\alpha \rightarrow \gamma\beta$.

- Given a set of functional dependencies

  F={ **A**→B, **B**→C, **DE**→A }.

- Prove, **AD** →B is true.
  - Since **A**→B, **AD**→BD (by **Augmentation**)
  - Since **B** $\subseteq$ BD, **BD**→B (by **Reflexivity**)
  - Since **AD**→BD and **BD**→B, <u>**AD**→B</u> (by **Transitivity**)

Please give the formal prove.

Done!

Question 1c

Armstrong's axioms

1. **Reflexivity** - if $\beta \subseteq \alpha$, then $\alpha \rightarrow \beta$.
2. **Transitivity** - if $\alpha \rightarrow \beta$ and $\beta \rightarrow \gamma$, then $\alpha \rightarrow \gamma$.
3. **Augmentation** - if $\alpha \rightarrow \beta$, then $\gamma \alpha \rightarrow \gamma\beta$.

- Given a set of functional dependencies

$$F=\{ A \rightarrow B, B \rightarrow C, DE \rightarrow A \}.$$

- Prove, **DE** → **ABC** is true.
- **Think in this way…**
  - We have **DE**→**A**, can we show that **A**→**ABC**?
    - We have **A**→**B**, and therefore **A**→**AB** (by **Augmentation**)
    - Can we show that **AB**→**ABC**?

      Since **B**→**C**, **AB**→**ABC** (by **Augmentation**)
  - So we now have : **DE**→**A**, **A**→**AB** , **AB**→**ABC** , done ☺ !!!

Question 1c

Armstrong's axioms

1. **Reflexivity** - if $\beta \subseteq \alpha$, then $\alpha \rightarrow \beta$.
2. **Transitivity** - if $\alpha \rightarrow \beta$ and $\beta \rightarrow \gamma$, then $\alpha \rightarrow \gamma$.
3. **Augmentation** - if $\alpha \rightarrow \beta$, then $\gamma\alpha \rightarrow \gamma\beta$.

- Given a set of functional dependencies

$$F=\{ A \rightarrow B,\ B \rightarrow C,\ DE \rightarrow A \}.$$

- Prove, $DE \rightarrow ABC$ is true.
  - Since $A \rightarrow B$, $A \rightarrow AB$ (by **Augmentation**)
  - Since $B \rightarrow C$, $AB \rightarrow ABC$ (by **Augmentation**)
  - Since $A \rightarrow AB$ and $AB \rightarrow ABC$, $A \rightarrow ABC$ (by **Transitivity**)
  - Since $DE \rightarrow A$ and $A \rightarrow ABC$, $\underline{DE \rightarrow ABC}$ (by **Transitivity**)

Please give the formal prove.

Done!

Armstrong's Axioms

- 3 basic axioms.
  - 1. Reflexivity - if $\beta \subseteq \alpha$, then $\alpha \rightarrow \beta$.
  - 2. Transitivity - if $\alpha \rightarrow \beta$ and $\beta \rightarrow \gamma$, then $\alpha \rightarrow \gamma$.
  - 3. Augmentation - if $\alpha \rightarrow \beta$, then $\gamma \alpha \rightarrow \gamma\beta$.
- 3 more axioms to help easier prove!
  - 4. Union - if $\alpha \rightarrow \beta$ and $\alpha \rightarrow \gamma$, then $\alpha \rightarrow \beta\gamma$.
  - 5. Decomposition - if $\alpha \rightarrow \beta\gamma$, then $\alpha \rightarrow \beta$ and $\alpha \rightarrow \gamma$.
  - 6. Pseudo-transitivity - if $\alpha \rightarrow \beta$ and $\gamma\beta \rightarrow \delta$, then $\alpha\gamma \rightarrow \delta$.

Armstrong's Axioms

R

A	B	C	D	E
1	5	2	5	4
1	4	3	2	3
3	4	3	2	2
4	2	4	1	4
4	1	4	1	4

A → B	✗	AB → AD	✓
B → C	✓	AC → CE	✓
C → D	✓	A → E	✗
AB → A	✓	**B → CD**	✓
B → D	✓		

4. Union - if α → β and α → γ, then α → βγ.

Think in this way…

If B → C, then **B → BC** is also true **(by augmentation)**

If B → D, then **BC → CD** is also true **(by augmentation)**

Therefore, with **B → BC** and **BC → CD ,**

B → CD is also true **(by transitivity).**

Armstrong's Axioms

R

A	B	C	D	E
1	5	2	5	4
1	4	3	2	3
3	4	3	2	2
4	2	4	1	4
4	1	4	1	4

A → B ✗

B → C ✓

C → D ✓

AB → A ✓

B → D ✓

AB → AD ✓

AC → CE ✓

A → E ✗

B → CD ✓

AC → C ✓

AC → E ✓

4. Union - if α → β and α → γ, then α → βγ.

5. Decomposition - if α → βγ, then α → β and α → γ.

Think in this way…

CE → C and CE → E are always true **(by reflexivity)**

Therefore, given **AC → CE,**

AC → C and AC → E are also true **(by transitivity).**

Armstrong's Axioms

R

A	B	C	D	E
1	5	2	5	4
1	4	3	2	3
3	4	3	2	2
4	2	4	1	4
4	1	4	1	4

A → B	✗	AB → AD	✓	AC → C	✓
B → C	✓	**AC → CE**	✓	AC → E	✓
C → D	✓	A → E	✗	**AB → CE**	✓
AB → A	✓	B → CD	✓		
B → D	✓				

4. Union - if α → β and α → γ, then α → βγ.

5. Decomposition - if α → βγ, then α → β and α → γ.

6. Pseudo-transitivity - if α → β and γβ → δ, then αγ → δ.

Think in this way…

If **B** → **C**, then **AB** → **AC** is true **(by augmentation)**

Therefore, given **AC** → **CE**,

AB → **CE** is also true **(by transitivity).**

Question 2

- Derive the following rules with Armstrong's axioms and the additional rules.

Armstrong's axioms

1. **Reflexivity** - if $\beta \subseteq \alpha$, then $\alpha \rightarrow \beta$.
2. **Transitivity** - if $\alpha \rightarrow \beta$ and $\beta \rightarrow \gamma$, then $\alpha \rightarrow \gamma$.
3. **Augmentation** - if $\alpha \rightarrow \beta$, then $\gamma\alpha \rightarrow \gamma\beta$.
4. **Union** - if $\alpha \rightarrow \beta$ and $\alpha \rightarrow \gamma$, then $\alpha \rightarrow \beta\gamma$.
5. **Decomposition** - if $\alpha \rightarrow \beta\gamma$, then $\alpha \rightarrow \beta$ and $\alpha \rightarrow \gamma$.
6. **Pseudo-transitivity** - if $\alpha \rightarrow \beta$ and $\gamma\beta \rightarrow \delta$, then $\alpha\gamma \rightarrow \delta$.

  - **a)** If **A**→**E**, **A**→**D** and **E**→**B** then **A**→**BD**.
  - **b)** If **M**→**J** and **JY**→**RC** then **MY**→**R**.
  - **c)** If **L**→**IJ** and **J**→**KH** then **L**→**KH**.

Question 2a

- Derive the following rules with Armstrong's axioms and the additional rules.

Armstrong's axioms

1. **Reflexivity** - if $\beta \subseteq \alpha$, then $\alpha \rightarrow \beta$.
2. **Transitivity -** if $\alpha \rightarrow \beta$ and $\beta \rightarrow \gamma$, then $\alpha \rightarrow \gamma$.
3. **Augmentation -** if $\alpha \rightarrow \beta$, then $\gamma \alpha \rightarrow \gamma\beta$.
4. **Union -** if $\alpha \rightarrow \beta$ and $\alpha \rightarrow \gamma$, then $\alpha \rightarrow \beta\gamma$.
5. **Decomposition -** if $\alpha \rightarrow \beta\gamma$, then $\alpha \rightarrow \beta$ and $\alpha \rightarrow \gamma$.
6. **Pseudo-transitivity -** if $\alpha \rightarrow \beta$ and $\gamma\beta \rightarrow \delta$, then $\alpha\gamma \rightarrow \delta$.

- Prove, if **A→E**, **A→D** and **E→B** then **A→BD**.
  - Since **A→E** and **E→B**, **A→B** (by **Transitivity**)
  - Since **A→B** and **A→D**, <u>**A→BD**</u> (by **Union**)

Question 2b

- Derive the following rules with Armstrong's axioms and the additional rules.

Armstrong's axioms

1. **Reflexivity** - if $\beta \subseteq \alpha$, then $\alpha \rightarrow \beta$.
2. **Transitivity** - if $\alpha \rightarrow \beta$ and $\beta \rightarrow \gamma$, then $\alpha \rightarrow \gamma$.
3. **Augmentation** - if $\alpha \rightarrow \beta$, then $\gamma \alpha \rightarrow \gamma\beta$.
4. **Union** - if $\alpha \rightarrow \beta$ and $\alpha \rightarrow \gamma$, then $\alpha \rightarrow \beta\gamma$.
5. **Decomposition** - if $\alpha \rightarrow \beta\gamma$, then $\alpha \rightarrow \beta$ and $\alpha \rightarrow \gamma$.
6. **Pseudo-transitivity** - if $\alpha \rightarrow \beta$ and $\gamma\beta \rightarrow \delta$, then $\alpha\gamma \rightarrow \delta$.

- Prove, if **M→J** and **JY→RC** then **MY→R**.
  - Since **M→J** and **JY→RC**, **MY→RC** (by **Pseudo-transitivity**)
  - Since **MY→RC**, **MY→R** (by **Decomposition**)

Question 2c

- Derive the following rules with Armstrong's axioms and the additional rules.

Armstrong's axioms

1. **Reflexivity** - if $\beta \subseteq \alpha$, then $\alpha \rightarrow \beta$.
2. **Transitivity** - if $\alpha \rightarrow \beta$ and $\beta \rightarrow \gamma$, then $\alpha \rightarrow \gamma$.
3. **Augmentation** - if $\alpha \rightarrow \beta$, then $\gamma\alpha \rightarrow \gamma\beta$.
4. **Union** - if $\alpha \rightarrow \beta$ and $\alpha \rightarrow \gamma$, then $\alpha \rightarrow \beta\gamma$.
5. **Decomposition** - if $\alpha \rightarrow \beta\gamma$, then $\alpha \rightarrow \beta$ and $\alpha \rightarrow \gamma$.
6. **Pseudo-transitivity** - if $\alpha \rightarrow \beta$ and $\gamma\beta \rightarrow \delta$, then $\alpha\gamma \rightarrow \delta$.

- Prove, if **L**→**IJ** and **J**→**KH** then **L**→**KH**.
  - Since **L**→**IJ**, **L**→**I** and **L**→**J** (by **Decomposition**)
  - Since **L**→**J** and **J**→**KH** , **L→KH** (by **Transitivity**)

Attribute set closure

Attribute set closure α^+

- Given a set F of FDs and a set of attributes α.
- The **closure of α** (denoted as α^+) is the set of attributes that can be **functionally determined by α**.

$$F = \{A \rightarrow B, B \rightarrow C\}$$

Attribute set closure of A.

$$\{A\}^+ = \{A, B, C\}$$

1. A → A is always true (by **Reflexivity**).
2. A → B is given in F.
3. A → C is derived from F:
Given A → B and B → C, A → C is also true (by **Transitivity**).

Attribute set closure α^+

- Given a set F of FDs and a set of attributes α.
- The **closure of α** (denoted as **α^+**) is the set of attributes that can be **functionally determined by α**.

$$F = \{A \rightarrow B, B \rightarrow C\}$$

Attribute set closure of A.

$$\{A\}^+ = \{A, B, C\}$$

$$\{B\}^+ = \{B, C\}$$

$$\{C\}^+ = \{C\}$$

$$\{A,B\}^+ = \{A, B, C\}$$

Note that we only consider **single attribute**, not attribute sets (so we do not have AB, ABC, AC…etc in $\{A,B\}^+$).

Computing α^+

result = **α.**
while (changes to ***result***){
    for each β → γ in **F** {
        if (**β** ⊆ ***result***){
            result = ***result*** ∪ **γ.**
        }
    }
}

The **attribute_closure()** algorithm

Simply speaking, we apply the **Transitivity** rule again and again to find all the attributes that are functionally determined by α**.**

Computing α^+

```
result = α
while (changes to result ){
   for each β → γ in F {
      if (β ⊆ result){
         result = result ∪ γ
      }
   }
}
```

The **attribute_closure()** algorithm

Reflexivity rule

- *The reflexivity rule states that $A \rightarrow A$ must be true.*

$F = \{A \rightarrow B,\ B \rightarrow C\}$

$\alpha = A$

Input to the **attribute_closure()** algorithm

$result = \{A\}$

Output of the **attribute_closure()** algorithm

Computing α^+

```
result = α
while (changes to result ){
   for each β → γ in F {
      if (β ⊆ result){
         result = result ∪ γ
      }
   }
}
```

The **attribute_closure()** algorithm

Transitivity rule

- ***We find the attributes that can be functionally determined by A, so we search for the rules in the format A → sth.***

$F = \{A \to B, B \to C\}$

$\alpha = A$

Input to the **attribute_closure()** algorithm

$result = \{ A \}$

Output of the **attribute_closure()** algorithm

Computing α⁺

```
result = α
while (changes to result ){
   for each β → γ in F {
      if (β ⊆ result){
         result = result ∪ γ
      }
   }
}
```

The **attribute_closure()** algorithm

Discover B

- *Since A → B is in F, we know that B is functionally determined by A.*

F = {A → B, B → C}

α = A

Input to the **attribute_closure()** algorithm

result = { A, B }

Output of the **attribute_closure()** algorithm

Computing α^+

```
result = α
while (changes to result ){
    for each β → γ in F {
        if (β ⊆ result){
            result = result ∪ γ
        }
    }
}
```

The **attribute_closure()** algorithm

Repeat until no more change

- ***In the next iteration, we consider the set of FDs in the format B → sth.***

Therefore, $\{A\}^+ = \{A, B, C\}$.

$F = \{A \rightarrow B, B \rightarrow C\}$

$\alpha = A$

Input to the **attribute_closure()** algorithm

$result = \{A, B, C\}$

Output of the **attribute_closure()** algorithm

Use of α^+

- **Testing for superkey: α is a super key of R iff α^+ contains all attributes of R.**
- **Check if the decomposition of a relation is dependency preserving or not.**
- **Calculate FD closure F^+, which is an important tool in database normalization (e.g., The Boyce-Codd normal form BCNF.)**

Question 3

- Given a relation R(A, B, C, D, E) and functional dependencies F={**C**$\rightarrow$**D**, **AC**$\rightarrow$**BE**, **D**$\rightarrow$**A**}.
  Prove that C is a candidate key of R.

Hints:

To prove C is a candidate key of R, we need to:

1) Prove that C is a **superkey.** **(How? Answer in the previous slide.)**
2) Prove that C is minimum (no subset of C is a **superkey**).

Ok! First I need to show that **$\{C\}^+$ = {A,B,C,D,E}.**
This implies **ALL** attributes are functionally determined by C, which means C is a **superkey**.

Question 3

- Given a relation R(A, B, C, D, E) and functional dependencies F={**C→D**, **AC→BE**, **D→A**}.
  Prove that C is a candidate key of R.

$\{C\}^+$ contains A ?

Since **C**→**D** and **D**→**A**, C→A (by **Transitivity**)

$\{C\}^+$ contains B ?

This is essentially asking if **C** →**A** is true or not, so we have the prove the FD **C** →**A** here.

$\{C\}^+$ contains C ?

$\{C\}^+$ contains D ?

$\{C\}^+$ contains E ?

If the answers to all these questions are **YES**, then C is a **superkey**.

Question 3

- Given a relation R(A, B, C, D, E) and functional dependencies F={C→D, AC→BE, D→A}.
  Prove that C is a candidate key of R.

$\{C\}^+$ contains A ?

Since C→D and D→A, C→A (by **Transitivity**)

$\{C\}^+$ contains B ?

Since C→A, C→AC (by **Augmentation**)

Since C→AC and AC→BE, C→BE (by **Transitivity**)

Since C→BE, C→B (by **Decomposition**)

$\{C\}^+$ contains C ?

$\{C\}^+$ contains D ?

Since C→D, $\{C\}^+$ contains D.

$\{C\}^+$ contains E ?

Since C→BE, C→E (by **Decomposition**)

Finally, since $\{C\}^+$ contains all attributes of R, C is a superkey of R.
C is a single attribute, it is a candidate key.

Done!

FD closure

FD closure F^+

- The set of **ALL functional dependencies that can be logically implied by F** is called the closure of F (or F^+)

- To compute $\mathbf{F^+}$ in a relation **R**:

This is the attribute set closure.

Step 1. Treat every subset of **R** as $\boldsymbol{\alpha}$.
Step 2. For every $\boldsymbol{\alpha}$, compute $\boldsymbol{\alpha}^+$.
Step 3. Use $\boldsymbol{\alpha}$ as LHS, and generate an FD for every subset of $\boldsymbol{\alpha}^+$ on RHS.

The **fd_closure()** algorithm

Question 4

- Given a relation R(N, S, P) and the functional dependencies F = {N→S, N→P} find the FD closure F^+.

	N	S	P	NS	NP	SP	NSP

Step 1. Treat every subset of **R** as **α**.

Question 4

- Given a relation R(N, S, P) and the functional dependencies F = {**N**→**S**, **N**→**P**} find the FD closure F^+.

	N	S	P	NS	NP	SP	NSP
Attribute set closure	{N,S,P}						

Step 2. For every **α**, compute **α**$^+$.

To find the attribute set closure {N}, use the attribute_closure() algorithm

1. ***result*** = {N}
2. Consider the FDs with **N** →**S**, **N** →**P**, add S and P into ***result.***
3. ***result*** = {N,S,P}

Question 4

- Given a relation R(N, S, P) and the functional dependencies F = {**N**→**S**, **N**→**P**} find the FD closure F^+.

	N	S	P	NS	NP	SP	NSP
Attribute set closure	{N,S,P}	{S}	{P}	{N,S,P}	{N,S,P}	{S,P}	{N,S,P}

Step 2. For every **α**, compute **α**$^+$.

To find the attribute set closure {N}, use the attribute_closure() algorithm

1. ***result*** = {N}
2. Consider the FDs with **N** →**S**, **N** →**P**, add S and P into ***result.***
3. ***result*** = {N,S,P}

Question 4

- Given a relation R(N, S, P) and the functional dependencies F = {**N→S**, **N→P**} find the FD closure F^+.

	N	S	P	NS	NP	SP	NSP
Attribute set closure	{N,S,P}	{S}	{P}	{N,S,P}	{N,S,P}	{S,P}	{N,S,P}
FD	N→N N→S N→P N→NS N→NP N→SP N→NSP						

Step 3. Use **α** as LHS, and generate an FD for every subset of **α**$^+$ on RHS.

Question 4

- Given a relation R(N, S, P) and the functional dependencies F = {N→S, N→P} find the FD closure F^+.

	N	S	P	NS	NP	SP	NSP
Attribute set closure	{N,S,P}	{S}	{P}	{N,S,P}	{N,S,P}	{S,P}	{N,S,P}
FD	N→N N→S N→P N→NS N→NP N→SP N→NSP	S→S	P→P	NS→N NS→S NS→P NS→NS NS→NP NS→SP NS→NSP	NP→N NP→S NP→P NP→NS NP→NP NP→SP NP→NSP	SP→S SP→P SP→SP	NSP→N NSP→S NSP→P NSP→NS NSP→NP NSP→SP NSP→NSP

Summary

- **The following concepts will be used in the discussions of database normalization**
  - Functional dependency.
  - FD closure.
  - Attribute set closure.
- **We would like to achieve the followings when we design the database schema**
  - No information loss
  - No redundancy
  - Preserve functional dependencies in individual relations

END

COMP3278A

Introduction to Database Management Systems

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